\[ x_1 = 1, \quad \begin{pmatrix} 2^{-1} & -1 & 0 \\ -1 & 2^{-1} & 0 \\ 0 & 0 & 4^{-1} \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 2_1 \end{pmatrix} = 0 = 0 \begin{pmatrix} x_1 \\ y_1 \\ 2_1 \end{pmatrix} = 0 \]

\[ T_1 = \frac{ML^2}{36} \text{ about } a \parallel (i) \]
\[ T_2 = \frac{ML^2}{12} \text{ about } b \parallel (j) \]
\[ T_3 = \frac{ML^2}{9} \text{ about } c \parallel (k) \]

Euler eq's of motion

Angular momentum:
\[ \frac{d}{dt} \begin{pmatrix} L \\ \omega \end{pmatrix} = \begin{pmatrix} N \\ \frac{d}{dt} \begin{pmatrix} L \\ \omega \end{pmatrix} \end{pmatrix} + \omega \times L \]

(true also for non-constant \( \omega \))
with \( L_i = \begin{pmatrix} I_i \omega_i \end{pmatrix} \) (in co-ordinate system of principal axes, \( \bar{T}_i = 0 \))

\[
\begin{align*}
\bar{T}_1 \omega_1 &= -\omega_2 \omega_3 (\bar{T}_2 - \bar{T}_3) = \bar{N}_1 \\
\bar{T}_2 \omega_2 &= -\omega_3 \omega_1 (\bar{T}_3 - \bar{T}_1) = \bar{N}_2 \\
\bar{T}_3 \omega_3 &= -\omega_1 \omega_2 (\bar{T}_1 - \bar{T}_2) = \bar{N}_3
\end{align*}
\]
Euler's of motion of rigid body with fixed point or COM

In case of COM; COM eq's of motion are totally independent!

Torque-free motion

\[
\begin{align*}
I_1 \omega_1 &= \omega_2 \omega_3 (I_2 - I_3) \\
I_2 \omega_2 &= \omega_3 \omega_1 (I_3 - I_1) \\
I_3 \omega_3 &= \omega_1 \omega_2 (I_1 - I_2)
\end{align*}
\]

Remember: 
\[
\varepsilon = \frac{\omega}{\sqrt{I_1}} = \frac{\omega}{\sqrt{2T}}
\]

Define: 
\[
F(\varepsilon) = \varepsilon \cdot I \varepsilon = \sum \varepsilon_i^2 I_i
\]

where \(F(\varepsilon) = 1\) in inertia ellipsoid.

=\# if axes of motion change, \(\varepsilon\) changes accordingly.

The tip (of the \(\varepsilon\)-vector) always defines a point on inertia ellipsoid!
\[ \Rightarrow \mathbf{\nabla}_s \mathbf{T} = \mathbf{n} \quad \text{surface of ellipsoid} \]

also:  \[ \mathbf{\nabla}_s \mathbf{T} = 2 \mathbf{I} \mathbf{n} = 2 \frac{\mathbf{I} \mathbf{n}}{\sqrt{2T}} = \sqrt{\frac{2}{T}} \mathbf{n} = \cos \theta \]

\[ \Rightarrow \mathbf{n} \text{ will always move such that the normal to the ellipsoid (at } \mathbf{n}) \parallel \mathbf{L} \]

also:  \[ \frac{\mathbf{n} \cdot \mathbf{L}}{\mathbf{L}} = \frac{\mathbf{L} \cdot \mathbf{L}}{\sqrt{2T}} = \frac{2T}{\sqrt{2T}} = \frac{\sqrt{2T}}{2} = \cos \theta \]

projection of \( \mathbf{n} \) along \( \mathbf{L} \) 

(\[ \Rightarrow \text{distance of tangential plane to midpoint of ellipsoid} \] = constant) 

\[ \Rightarrow \text{tangential plane through } \mathbf{n} \text{ is invariable} \rightarrow \text{invariable plane} \]

pohlade

herpolade rolls with origin at constant height without slipping along invariable plane.

line traced out on ellipsoid = "pohlade"

plane = "herpolade"
"The polepole rolls without slipping on the heptahode lying in the riverable plane."

**Symmetric cusp elliptical:**
- polepole, heptahode = cycles
- prolate \( I_3 < I_1 = I_2 \) (football shaped)
- traces cone in body-fixed system
- space-fixed body - cone outside of space cone

\[-oblate (I_3 > I_1 = I_2)\]
- both cases: \( \text{w}(g) \) precesses about symmetry axis of body
**Symmetric case** \((I_1 > I_2 > I_3)\)

\[
T = \frac{L_1^2}{2I_1} + \frac{L_2^2}{2I_2} + \frac{L_3^2}{2I_3}
\]

or

\[
1 = \frac{L_1^2}{2I_1T} + \frac{L_2^2}{2TI_2} + \frac{L_3^2}{2TI_3}
\]

also

\[
\frac{L_1^2}{L_2^2} + \frac{L_2^2}{L_3^2} + \frac{L_3^2}{L_1^2} = 1
\]

**Ellipsoidal with half axes** \(\sqrt{2TI_1}\) in \(L\)-space

**Path of \(L\) along intersection of both!**

with \(\sqrt{2TI_1} > L > \sqrt{2TI_3}\)

How can asymmetric body move?

Steady rotation (i.e. \(\omega = \text{const}\)) only possible about one of the principal axes!

Proof: From Euler's equation (with \(i = 0\))

\[
0 = \omega_2 \omega_3 (I_2 - I_3) + \omega_3 \omega_1 (I_3 - I_1)
= \omega_1 \omega_2 (I_1 - I_2)
\]
If $I_1 + I_2 + I_3 = -I$, at least two of $I_1, I_2, I_3$ have to be $0$.

Stability of these rotations:

1. Steady motion around $z$ ($L = \left( \begin{array}{c} L_1 \\ L_2 \\ L_3 \end{array} \right)$, $L^2 = 2I$)
   - Small deviation $\Rightarrow L$ slightly larger
   - Intersection results in small closed orbits around $z$-axis (cf. Fig. 5.5)
   - Motion is stable

2. Steady motion around $x$ ($L = \left( \begin{array}{c} L_1 \\ L_2 \\ 0 \end{array} \right)$, $L^2 = 2I_x$)
   - Small deviation $\Rightarrow L$ slightly smaller
   - Intersection results in small closed orbits around $x$-axis (cf. Fig. 5.5)
   - Motion is stable

3. Steady motion around $y$ ($L = \left( \begin{array}{c} L_1 \\ 0 \\ L_3 \end{array} \right)$, $L^2 = 2I_y$)
   - Small deviation $\Rightarrow L$ slightly larger or smaller
   - Intersection results in unstable (cf. Fig. 5.5)
   - Motion is unstable
symmetric case e.g. $\bar{I}_1 = \bar{I}_2$

1) $\bar{I}_2 \omega_1 = (\bar{I}_1 - \bar{I}_3) \omega_2 \omega_3$
2) $\bar{I}_1 \omega_2 = (\bar{I}_3 - \bar{I}_1) \omega_3 \omega_1$
3) $\bar{I}_3 \omega_3 = 0 \implies \omega_3 = \text{const}$

\[
\begin{align*}
\omega_1 &= -\Omega \omega_2 \\
\omega_2 &= \Omega \omega_3 \implies \Omega = \omega_3 \frac{\bar{I}_3 - \bar{I}_1}{\bar{I}_1} \\
\omega_1 &= A \cos \Omega t \\
\omega_2 &= A \sin \Omega t
\end{align*}
\]

describes precession about $z$-axis (and

\[ w = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2} = \text{const} \]

with

\[ T = \frac{1}{2} \bar{I}_1 A^2 + \frac{1}{2} \bar{I}_3 \omega_3^2 \]

solve for

\[ L^2 = \bar{I}_1 A^2 + \bar{I}_3 \omega_3^2 \]

\[ \bar{I}_1 \bar{I}_3 - \bar{I}_1 = 0.00327 \]

(Precession period of about 10 months rel. to Earth's $z$-axis!)

\[ \implies \text{"Chandler wobble"?} \]