Show all your work and indicate your reasoning in order to receive the credit. Present your answers in *low-entropy* form.

Name: ______________________________

Date: ______________________________

Question:	1	2	3	4	Total
Points:	40	40	10	10	100
Score:					

1. Consider the following differential equation

$$\frac{d^2y}{dt^2} + t\,\frac{dy}{dt} + t^2 y = 0.$$

Find the series solution about $x = 0$.

(a) (5 points) Classify the point $x = 0$ for the equation as ordinary, regular singular, or irregular singular.

(b) (10 points) Write down the solution as a series about $x = 0$ and, if applicable, determine the indicial equation and find the corresponding roots.

(c) (15 points) Find the recurrence relation that determines the coefficients in a series solution to this equation about $x = 0$.

(d) (10 points) Find the first four non-zero terms of the series solution of the following initial value problem for the equation above:

$$y(0) = 1, \quad y'(0) = 0.$$

2. Consider the following differential equation

$$t^2\,\frac{d^2y}{dt^2} + t\,\frac{dy}{dt} + (t-1)\,y = 0.$$

Find the series solution about $x = 0$.

(a) (5 points) Classify the point $x = 0$ for the equation as ordinary, regular singular, or irregular singular.

(b) (10 points) Write down the solution in a series form and, if applicable, determine the indicial equation and find the corresponding roots.

(c) (15 points) Find the recurrence relation that determines the coefficients in a series solution.

(d) (10 points) Find the first four non-zero terms of the series solution of the following initial value problem:

$$y(0) = 0, \quad y'(0) = 1.$$

3. (10 points) Find the general solution of the following equation:

$$(t-1)^2 \frac{d^2y}{dt^2} + 4(t-1)\frac{dy}{dt} + 2y = 0, \quad t > 1.$$

Hint: introdice a new independent variable $x = t - 1$.

4. Consider the differential equation

$$x^3 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

(a) (5 points) Find and classify the finite singular point of the equation.

(b) (5 points) Find the Wronskian of the equation. Use the fact that $y_1(x) = x$ is a solution to find a second independent solution $y_2(x)$.