Show all your work and indicate your reasoning in order to receive the most credit. Present your answers in *low-entropy* form. Write your name on the problems page and staple it together with your solutions.

Name: ______________________________

Date: ______________________________

Question:	1	2	3	Total
Points:	25	25	20	70
Score:				

Geometry of palnar curves

1. Start your derivations from the equations describing a planar curve using the arc length, s, and the elevation angle, $theta$, that we used in class.

   (a) (15 points) Show that the radius of curvature, R. of a curve $y = y(x)$ is

$$\frac{1}{R} = \frac{y''(x)}{\left[1 + y'(x)^2\right]^{\frac{3}{2}}} \tag{1}$$

   (b) (10 points) Use Eq. (1) to Calculate the radius of curvature of a cycloid specified by the following *parametric* equations:

$$\begin{aligned} x(\theta) &= R\,(\theta - \sin(\theta)) \\ y(\theta) &= -R\,(1 - \cos(\theta)) \end{aligned}$$

Hydrostatics

2. (25 points) Obtain an expression for the pressure at the center of self-gravitating spherical star with the following density at a distance r from the center:

$$\rho(r) = \rho_0 \left(1 - \beta r^2\right)$$

   Hint: find the radius of the star; find the acceleration of gravity due to the star at the distance r from its center.

Elastostatics

3. During emergency repairs on the starliner "Axiom" a swimming pool (of the depth H) had been filled up to the brim with a soft elastic material of density ρ and Lamé elastic constants λ and μ. When the normal gravity was restored, a pothole was formed at the place of the former pool.

   (a) (10 points) What is the depth of the pothole?

   (b) (10 points) Lamé elastic constants can be expressed as following.

$$\lambda = \frac{\nu E_Y}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E_Y}{2(1+\nu)},$$

   where E_Y and ν are the material's Young's modulus and Poisson's ratio. What should be the Poisson's ratio of the elastic pool filling so that no pothole is formed? (Note: for isotropic materials $0 \le \nu \le \frac{1}{2}$.)