Show all your work and indicate your reasoning in order to receive the most credit. Present your answers in *low-entropy* form.

Name: _____

Date: _____

Question:	1	2	3	4	Total
Points:	30	30	30	30	120
Score:					

Viscous flow

1. (30 points) If you dip a cylinder into syrup, withdraw it, and hold it horizontally, you can prevent the syrup from draining off by rotating the cylinder. Determine the maximal load of the syrup per unit length of the cylinder as a function of the radius of the cylinder a, its angular velocity of rotation ω, syrup viscosity ν, its density ρ, and acceleration of gravity g. Assume that the syrup film thickness $h \ll a$.

2. (30 points) A vertical pipe is filled with a viscous incompressible fluid with the density ρ and viscosity ν. A long light cylinder (its density is much less than ρ) of radius R and length L, ($L \gg R$) is immersed co-axially into the pipe so that only a small gap of width $h \ll R$ is formed between their lateral surfaces. Find the terminal velocity of the cylinder.

Surface waves

3. (30 points) One of the striking features of the wake pattern formed on the fluid surface behind a point source (e.g. a boat) moving with a constant velocity is two wake lines that together form the arms of a "V". The entire wake pattern moves as if it were attached to the source.

   Find the angle that each arm of the V makes with respect to the source trajectory.

Dynamics of ideal fluids

4. (30 points) A long cylinder of radius r and negligible mass (its density is small in comparison with the density of the surrounding fluid) is immersed co-axially in a vertical cylindrical container of radius R. The length of the cylinder, $L \gg r, R$. The container is filled with an ideal incompressible fluid. The cylinder is held inside the fluid by a long elastic spring (spring constant k) attached to the bottom of the container. Find the frequency of the small vertical oscillations of the cylinder about its equilibrium position.