

Answer all questions in the blue notebook provided. In order to receive the most credit show all your work and indicate your reasoning clearly.

1. Find the trace of the following tensor.

$$C_{ij} = \delta_{ij}\delta_{kl}\delta_{kl} + \delta_{ik}\delta_{jl}\delta_{kl} + \delta_{il}\delta_{jk}\delta_{kl},$$

where δ_{pq} is the Kronecker's delta, and the summation over repeated indices is assumed.

2. The displacement components for a body are as following.

$$\begin{aligned}u_1 &= \frac{\alpha}{2} X_1^2, \\u_2 &= \beta X_2 X_3, \\u_3 &= \gamma (X_1 X_3 + X_1^2),\end{aligned}$$

where X_1 , X_2 , and X_3 are material coordinates of the body; α , β , and γ are small dimensionless constants.

- (a) Find the strain tensor.
- (b) Given $\alpha = 10^{-4}$, find the parameters β and γ such that the deformation does not cause a volume change of the body.
- (c) Using the found values of α , β , and γ , find the relative extension of a linear element located (before the deformation) at the point (1,1,1) and oriented parallel to X_3 axis.
3. A stress tensor in a body is given by

$$\begin{pmatrix} 2 & -1 & 3 \\ \dots & 4 & 0 \\ \dots & \dots & -1 \end{pmatrix} \text{MPa}$$

- (a) Using the properties of the stress tensor find the missing elements.
- (b) Find the tension \vec{t} at a point on a plane whose normal is in the direction of $2\hat{e}_1 + 2\hat{e}_2 + \hat{e}_3$.