

In order to receive the most credit show all your work and indicate your reasoning clearly.

Problem 1. Photon sails collision

A photon sail is a (mostly hypothetical) form of spacecraft propulsion that uses the radiation pressure of light to accelerate material objects.

1. Velocity of a relativistic sail.

A photon sail of mass m is initially at rest (in the lab reference frame L). The photons from the accelerating light beam are absorbed by the sail. Find the following:

- (a) the mass of the sail as a function of its velocity;
- (b) the velocity of the sail as a function of the energy transferred to the sail by the laser beam.

2. Photon sails collision I.

Two identical photon sails, each of mass m , are initially at rest in the lab reference frame. Assume the line connecting the sails is the y axis. The sails are accelerated toward each other until their speeds in the lab frame reach 0.5. The sails move with constant velocities until they eventually collide and make one blob which is at rest. Find the mass of the blob.

3. Photon sails collision II. Momentum paradox.

Consider the setup of Part 2 observed from a reference frame S that moves with the velocity v along x axis. The following reasoning is incorrect:

The forces on the sails point in the y direction. Therefore, there is no change in the momentum of the sails in the x direction. But the mass of the resulting blob is greater than the sum of the initial sails masses (as you found in part 2). Hence, the speed of the resulting blob must be less than v (to keep p_x constant), so the whole apparatus slows down in the x direction.

State what is invalid in this reasoning. Consider the sails collision in S frame and show that the result is consistent with the result of Part 2 of the assignment.