Name: ____________________

Date: ____________________

Show all your work and indicate your reasoning in order to receive the most credit. Present your answers in *low-entropy* form. Write your name on the problems page and staple it together with your solutions.

Hint: to plot the numerical solution of a differential equation, e.g. to plot the solution of the differential equation from Problem 1 on the interval $0 \le x \le 20$, to compare it with your asymptotics, you may use the following commands:

```
sol = NDSolve[{y''[x]+y[x]+.1*y[x]^3 == 0, y[0]==1, y'[0]==0}, y[x],
              {x, 0, 20}]
Plot[Evaluate[y[x] /. sol], {x, 0, 20}]
```

Question:	1	2	3	Total
Points:	15	20	25	60
Score:				

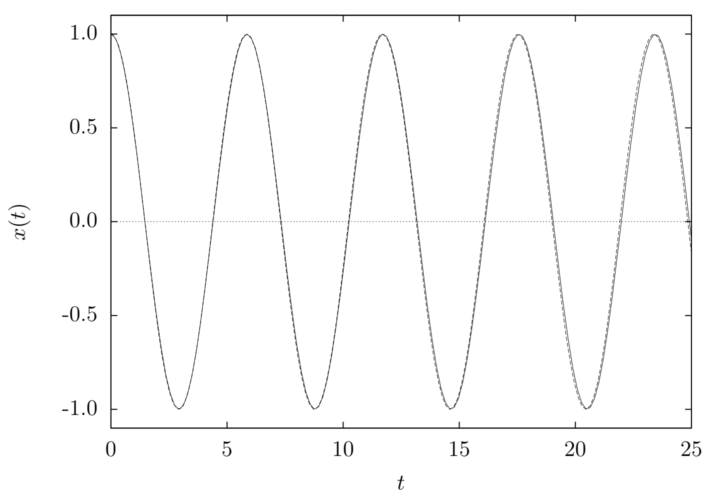
Method of averaging

1. (15 points) Find (a) the time dependence of the amplitude and (b) the frequency of the *Duffing oscillator*:

$$\ddot{x} + x + \epsilon x^3 = 0,$$

where ϵ is a small parameter, $\epsilon \ll 1$; $x(0) = a_0$, $\dot{x}(0) = 0$. Compare your analytic approximation with the numerical solution of the differential equation.

Figure 1: Typical solution of the Duffing equation from Problem 1 for small values of μ; $\mu = 0.2$ (solid line). The approximation obtained by the method of averaging is also shown (dashed line).



2. (20 points) Find the time dependence of the amplitude of an oscillator with "dry" friction:

$$\ddot{x} + \gamma\,\mathrm{sign}(\dot{x}) + x = 0,$$

where γ is a small parameter, $\gamma \ll 1$; $x(0) = a_0$, $\dot{x}(0) = 0$,

$$\mathrm{sign}(\alpha) = \begin{cases} 1, & \alpha > 0, \\ 0, & \alpha = 0, \\ -1 & \alpha < 0. \end{cases}$$

Compare your analytic approximation with the numerical solution of the differential equation.

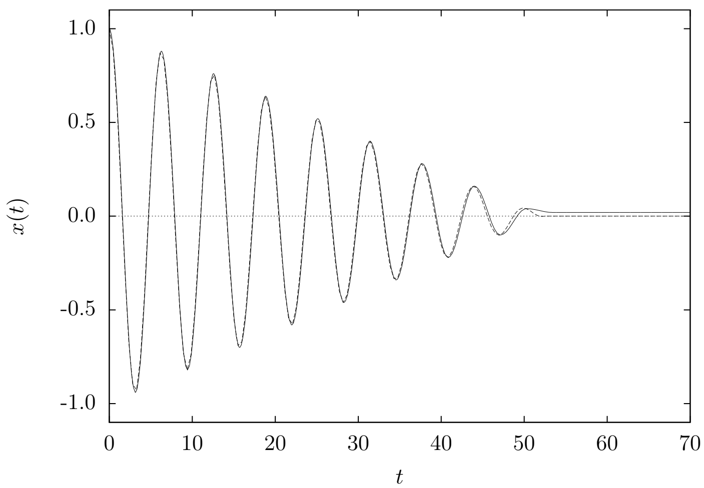


Figure 2: Typical solution of the dry friction equation from Problem 2 for small values of γ; $\gamma = 0.03$ (solid line). The approximation obtained by the method of averaging is also shown (dashed line).

3. (25 points) Find the solution of the following nonlinear differential equation:

$$\ddot{x} + \epsilon\dot{x}^5 + x = 0, \quad x(0) = x_0, \quad \dot{x}(0) = 0,$$

where ϵ is a small positive parameter. Compare your analytic approximation with the numerical solution of the differential equation.

Hint: show that

$$\overline{\sin^6(x)} \equiv \frac{1}{2\pi}\int_0^{2\pi} \sin^6(x)\mathrm{d}x = \frac{5}{16}$$

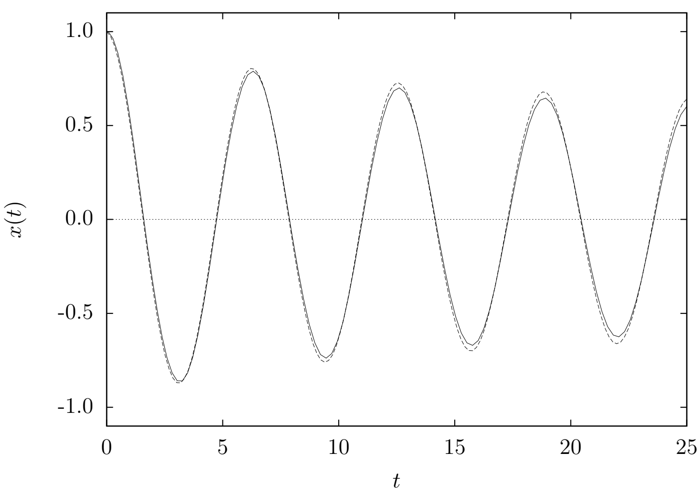


Figure 3: Typical solution of the nonlinear friction equation from Problem 3 for small values of μ; $\mu = 0.2$ (solid line). The approximation obtained by the method of averaging is also shown (dashed line).