Show all your work and indicate your reasoning in order to receive the most credit. You are welcome to discuss the problems with anyone you want (including your classmates and the instructor) but the solutions have to be executed and submitted individually.

1. (20pt) Evaluate the following sum:

$$S(x) = \sum_{n=1}^{\infty} \frac{\sin(nx)}{2^n n}. \tag{1}$$

On the same graph plot your answer and the sum of the first 5 terms in Eq. (1). Use the interval $-5. \leq x \leq 5.$ for plotting.

2. (40pt) An infinite hollow conducting circular cylinder of unit radius is cut into four equal parts by the planes $x = 0$, $y = 0$. (See Fig. 1, z axis is along the symmetry axis of the cylinder.) The segments in the first and third quadrant are maintained at potentials $+V_0$ and $-V_0$ respectively, and the segments in the second and fourth quadrant are maintained at zero potential. Find the potential $V(x, y)$ inside the cylinder.

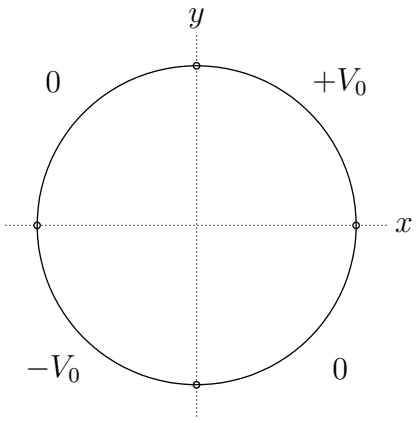

Figure 1: Four part cylinder. Small circles indicated non-conducting cuts.

Hint 1: Find the solution of the following simpler problems: the cylinder is cut into two equal parts by the plane $y = 0$, with the upper half maintained at potential $+V_0/2$ and the lower half maintained at potential $-V_0/2$. (See Fig. 2.) Use the superposition principle to solve the original problem.

Hint 2: In order to solve the problem of Hint 1, use the following conformal transformation

$$w = i\frac{1 - z}{1 + z} \tag{2}$$

that maps the interior of the cylinder's crossection onto the upper half of the w plane.

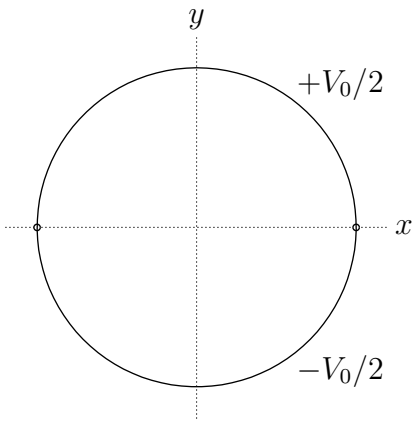

Figure 2: Auxiliary problem - two part cylinder.