Name: _____

Date: _____

Question:	1	2	3	Total
Points:	40	35	25	100
Score:				

1. Evaluate the following integral

$$I_n = \frac{\pi}{2} \int_0^1 x^n \cos\left(\frac{\pi}{2} x\right) \mathrm{d}x \tag{1}$$

for even $n = 0, 2, 4 \ldots 32$.

(a) (10 points) Integrate Eq. (1) by parts and obtain a relation between I_n and I_{n-2}. Evaluate I_0 analytically.

(b) (10 points) Write a C program that uses the recurrent relation as well as the initial condition obtained in (a) to evaluate I_2, I_4, $I_6 \ldots I_{32}$ numerically.

(c) (10 points) Describe the problem you run into when using the algorithm of part (b). What is the cause of the problem?

(d) (10 points) Modify the recurrence relation from part (a) to avoid the problem you discussed in part (c). What is a suitable initial condition? Write a C program that uses the modified recurrent relation to evaluate I_n numerically. Print side by side the results produces by the algorithms (b) and (d).

Enclose the printouts of your code, your Makefile, and the final program output.

2. (35 points) The **eight queens puzzle** is the problem of placing eight chess queens on an 8×8 chessboard so that no two queens attack each other. I.e., a solution requires that no two queens share the same row, column, or diagonal.

A brute force approach to the problem can be quite computationally expensive as there are

$$\frac{64 \cdot 63 \cdot 62 \cdot 61 \cdot 60 \cdot 59 \cdot 58 \cdot 57}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 4426165368$$

possible arrangements of eight queens on a 8×8 board.

By applying a simple rule that constrains each queen to a single column it is possible to reduce the number of possibilities to just

$$8^8 = 16777216$$

possible combinations.

Generating permutations further reduces the possibilities to just

$$8! = 40320,$$

which are then checked for diagonal attacks.

Write a C program that uses the permutation approach to print all possible solutions of the eight queens puzzle. Use the code from the class website as your starting point.

Submit the printout of your code, your Makefile, and one or two board configurations.

3. Imagine that as a part of the solution of a particular physical problem you need repeated calculations of the following expression:

$$\frac{\cos(x) + \frac{1}{2}\sin^2(x) - 1}{\sin^4(x)}. \tag{2}$$

for small x, $2^{-20} \leq x \leq 2^{-5}$.

(a) (5 points) Analytically find the limit

$$\lim_{x \to 0} \frac{\cos(x) + \frac{1}{2}\sin^2(x) - 1}{\sin^4(x)}.$$

(b) (10 points) Write a small C program (15–20 lines, not counting comments) that evaluates and prints expression Eq. (2) for $x = 2^{-5}, 2^{-6}, \ldots, 2^{-20}$. Compare the results with your analytic calculation. Why there is a difference?

(c) (10 points) Rewrite Eq. (2) into a form that is suitable for numerical calculations. Modify your program to print (with 14 figures after the decimal point, e.g using '%18.14f' format specifier) side by side x, the result given by Eq. (2), and the result of your modified expression.

Submit the printout of your code, your Makefile, final program output, explanation of the result in part (b) of the question.