

Name: _____

Date: _____

Question:	1	Total
Points:	30	30
Score:		

1. (30 points) Write a C program that implements the algorithm presented below.

5. An algorithm for π . We now present the details of our exponentially converging algorithm for calculating the digits of π . Twenty iterations will provide over two million digits. Each iteration requires about ten operations. The algorithm is very stable with all the operations being performed on numbers between $1/2$ and 7. The eighth iteration, for example, gives π correctly to 694 digits.

THEOREM 2. Consider the three-term iteration with initial values

$$\alpha_0 := \sqrt{2}, \quad \beta_0 := 0, \quad \pi_0 := 2 + \sqrt{2}$$

given by

$$(i) \quad \alpha_{n+1} := \frac{1}{2} (\alpha_n^{1/2} + \alpha_n^{-1/2}),$$

$$(ii) \quad \beta_{n+1} := \alpha_n^{1/2} \left(\frac{\beta_n + 1}{\beta_n + \alpha_n} \right),$$

$$(iii) \quad \pi_{n+1} := \pi_n \beta_{n+1} \left(\frac{1 + \alpha_{n+1}}{1 + \beta_{n+1}} \right).$$

Then π_n converges exponentially to π and

$$|\pi_n - \pi| \leq \frac{1}{10^{2^n}}.$$

Print the result of each iteration of the algorithm. Compare the results of your calculations with the value of the constant **M_PI** defined in **math.h** file. Make as many iterations as necessary to achieve the accuracy better than 10^{-10} . Provide printouts of your (formatted) C code, your makefile, and (nicely formatted) output of your program.