Name: ______________________________

Date: ______________________________

Question:	1	2	3	Total
Points:	50	5	5	60
Score:				

1. (50 points) One of the world's most extensively studied system of ordinary differential equations is the so called Lorenz system describing *Lorenz chaotic attractor*. The Lorenz equations were derived in 1963 as an approximation description of circulation in a shallow layer of fluid, heated uniformly from below and cooled uniformly from above. This fluid circulation is known as *Rayleigh-Benard convection*. The fluid is assumed to circulate in two dimensions (vertical and horizontal) with periodic rectangular boundary conditions. The partial differential equations modeling the system's stream function and temperature are subjected to the following approximation: the hydrodynamic fields are expanded in Fourier series, which are then truncated to a single term for the stream function and two terms for the temperature. This reduces the model equations to a set of three coupled, nonlinear ordinary differential equations.

$$\frac{dx}{dt} = \sigma(y - x), \tag{1}$$
$$\frac{dy}{dt} = x(\rho - z) - y, \tag{2}$$
$$\frac{dz}{dt} = xy - \beta z, \tag{3}$$

where t is time, x, y, and z make up the system state, and σ, ρ, and β are system parameters.

The solutions to this nonlinear system cannot be expressed in terms of other known functions; the equations must be solved numerically.

   1. Compute the solution with $x_0 = 0$, $y_0 = 1$, $z_0 = 0$ and $\sigma = 10$, $\rho = 28$, and $\beta = 8/3$. Use an integration algorithm that doesn't require Jacobian.
   2. Make plots of x, y, and z as functions of t, and a 3d phase plot.

Provide printouts of your (formatted) C code, your makefile, and (nicely formatted) output of your program.

2. (5 points) An adaptive ODE integrator . . .
   - ☐ is always the fastest method to solve an ODE
   - ☐ is always the most precise algorithm
   - ☐ adjusts the integration path to be parallel to the graph of the equation
   - ☐ adjusts the integration stepsize to keep the absolute and relative errors within requested limits
   - ☐ All of the above
   - ☐ None of the above

3. (5 points) In numerical analysis, the Euler method . . .
   - ☐ is a first-order numerical procedure for solving initial value problem for ordinary differential equations
   - ☐ is a third-order numerical procedure for solving initial value problem for ordinary differential equations
   - ☐ should always be the first method to try when solving ordinary differential equations
   - ☐ All of the above
   - ☐ None of the above