Name: ______________________________

Date: ______________________________

Question:	1	2	3	Total
Points:	50	5	5	60
Score:				

1. (50 points) A classical model in mathematical ecology is the Lotka-Volterra predator-prey model. Consider a simple ecosystem consisting of rabbits that have an infinite supply of food and foxes that prey on the rabbits for their food. This is modeled by a pair of nonlinear, 1st-order differential equations:

$$\frac{dr}{dt} = 2r - \alpha r f, \quad r(0) = r_0, \tag{1}$$
$$\frac{df}{dt} = -f + \alpha r f, \quad f(0) = f_0, \tag{2}$$

where t is time, $r(t)$ is the number of rabbits, $f(t)$ is the number of foxes, and α is a positive constant. If $\alpha = 0$, the two populations do not interact, the rabbits multiply, and the foxes die off from starvation. If $\alpha > 0$, the foxes encounter the rabbits with a probability that is proportional to the product of their numbers. Such an encounter results in a decrease in the number of rabbits and (for less obvious reasons) an increase in the number of foxes.

The solutions to this nonlinear system cannot be expressed in terms of other known functions; the equations must be solved numerically. It turns out that the solutions are always periodic, with a period that depends on the initial conditions. In other words, for any $r(0)$ and $f(0)$, there is a value $t = t_p$ when both populations return to their original values. Consequently, for all t, $r(t + t_p) = r(t)$, $f(t + t_p) = f(t)$.

   1. Compute the solution with $r_0 = 300$, $f_0 = 150$, and $\alpha = 0.01$. You should find that t_p is close to 5. Make two plots, one of r and f as functions of t and one a phase plane plot with r as one axis and f as the other.
   2. The point
   $$(r_0, f_0) = (\frac{1}{\alpha}, \frac{2}{\alpha})$$
   is a stable equilibrium point. If the populations have these initial values, they do not change. Numerically check this statement for $\alpha = \frac{1}{100}$.

For (a) and (b) use an integration algorithm that doesn't require Jacobian.

Provide printouts of your (formatted) C code, your makefile, and (nicely formatted) output of your program.

2. (5 points) A team of students is playng a game of Hanoi Tower with 20 disks. Estimate how long the game will take if they move on average one disk per second. For the purpose of the estimation assume that there are 2^6 seconds per minute, 2^6 minutes per hour, 2^4 hours per day, 2^5 days per month, 2^4 months per year.

   - ☐ Less than one minute
   - ☐ Between one minute and one hour
   - ☐ Between one hour and one day
   - ☐ Between one day and one month
   - ☐ Between one months and one year
   - ☐ Between one and four years
   - ☐ None of the above

3. (5 points) In numerical analysis, the shooting method is …

   - ☐ a method for solving a boundary value problem by reducing it to the solution of an initial value problem.
   - ☐ a method for solving an initial value problem by reducing it to the solution of a boundary value problem.
   - ☐ a method for solving an arbitrary differential equation by reducing it to the equation of a generalized projectile motion.
   - ☐ All of the above
   - ☐ None of the above