Physics 1501

Fall 2008

Mechanics, Thermodynamics, Waves, Fluids

Lecture 35: Oscillatory motion II
Recap: simple harmonic motion

• **Simple harmonic motion** (SHM) results when the force or torque that tends to restore equilibrium is directly proportional to the displacement from equilibrium.

  • The paradigm example is a mass $m$ on a spring of spring constant $k$.
    • The **angular frequency** $\omega = 2\pi$ for this system is
      $$\omega = \sqrt{\frac{k}{m}}$$
    • In SHM, displacement is a sinusoidal function of time:
      $$x(t) = A \cos \omega t$$
    • Any amplitude $A$ is possible.
      • In SHM, frequency doesn’t depend on amplitude.
Recap: quantities in simple harmonic motion

- Angular frequency, frequency, period:

\[
\omega = \sqrt{\frac{k}{m}} \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}}
\]

- Phase
  - Describes the starting time of the displacement-versus-time curve in oscillatory motion: \( x(t) = A\cos(\omega t + \phi) \)
Recap: velocity and acceleration in SHM

- Velocity is the rate of change of position:
  \[ v(t) = \frac{dx}{dt} = -\omega A \sin \omega t \]

- Acceleration is the rate of change of velocity:
  \[ a(t) = \frac{dv}{dt} = -\omega^2 A \cos \omega t \]
Recap: other simple harmonic oscillators

- The torsional oscillator
  - A fiber with torsional constant $\kappa$ provides a restoring torque.
  - Frequency depends on $\kappa$ and rotational inertia:
    $$\omega = \sqrt{\kappa / I}$$

- The pendulum
  - Simple pendulum
    - Point mass on massless cord of length $L$:
      $$\omega = \sqrt{g / L}$$
Energy in simple harmonic motion

- In the absence of nonconservative forces, the energy of a simple harmonic oscillator does not change.
- But energy is transferred back and forth between kinetic and potential forms.

\[
E = U + K
\]

- Total energy \( E \) is constant . . .
- while potential energy \( U \) and kinetic energy \( K \) oscillate.
Two different mass-spring systems are oscillating with the same amplitude. If one has twice as much total energy as the other, how does the spring constant of the more energetic system compare with that of the less energetic system?

A. It is the same.
B. It is half as great.
C. It is one-fourth as great.
D. It is twice as great.
E. It is four times as great.
Simple harmonic motion is ubiquitous!

- That’s because most systems near stable equilibrium have potential-energy curves that are approximately parabolic.
  - Ideal spring: \( U = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2 \)
  - Typical potential-energy curve of an arbitrary system:
Damped harmonic motion

- With nonconservative forces present, SHM gradually damps out.
  - Amplitude declines exponentially toward zero:
    \[ x(t) = Ae^{-bt/2m} \cos(\omega t + \phi) \]
  - For weak damping \( b \), oscillations still occur at approximately the undamped frequency \( \omega = \sqrt{k/m} \).
  - With stronger damping, oscillations cease.
    - **Critical damping** brings the system to equilibrium most quickly.

\[ \text{(a) Underdamped, (b) critically damped, and (c) overdamped oscillations.} \]
Resonance

- When a system is driven by an external force at near its natural frequency, it responds with large-amplitude oscillations.
  - This is the phenomenon of resonance.
  - The size of the resonant response increases as damping decreases.
  - The width of the resonance curve (amplitude versus driving frequency) also narrows with lower damping.

Resonance curves for several damping strengths; \( \omega_0 \) is the undamped natural frequency \( \sqrt{k/m} \).
Summary

- Oscillatory motion is periodic motion that results from a force or torque that tends to restore a system to equilibrium.

- In simple harmonic motion (SHM), the restoring force or torque is directly proportional to displacement.
  - The mass-spring system is the paradigm simple harmonic oscillator.
    \[ x(t) = A \cos \omega t \]
    \[ \omega = \sqrt{\frac{k}{m}} \]

- Damped harmonic motion occurs when nonconservative forces act on the oscillating system.

- Resonance is a high-amplitude oscillatory response of a system driven at near its natural oscillation frequency.