Physics 1501

Fall 2008

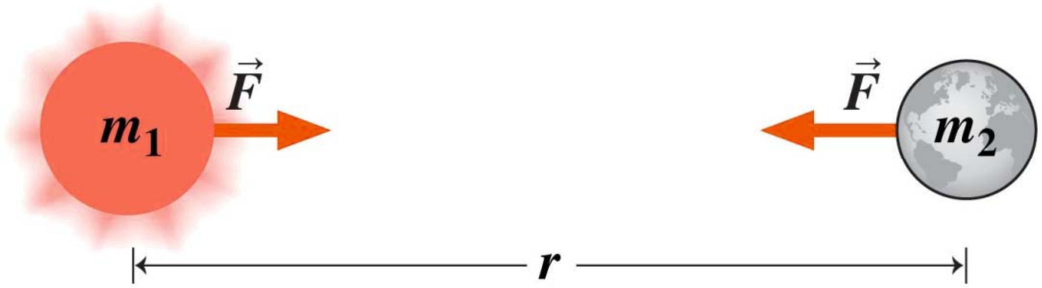
Mechanics, Thermodynamics, Waves, Fluids

Lecture 33: Gravity

Universal gravitation

- Introduced by Isaac Newton, the **Law of Universal Gravitation** states that any two masses m_1 and m_2 attract with a force F that is proportional to the product of their distances and inversely proportional to the distance r between them:

$$F = \frac{Gm_1 m_2}{r^2}$$



- Here $G = 6.67\times10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$ is the constant of universal gravitation.
- Strictly speaking, this law applies only to point masses. But Newton showed that it applies to spherical masses of any size, and it is a good approximation for any objects that are small compared with their separation.

question

Suppose the distance between two objects is cut in half. The gravitational force between them is …

A. doubled.

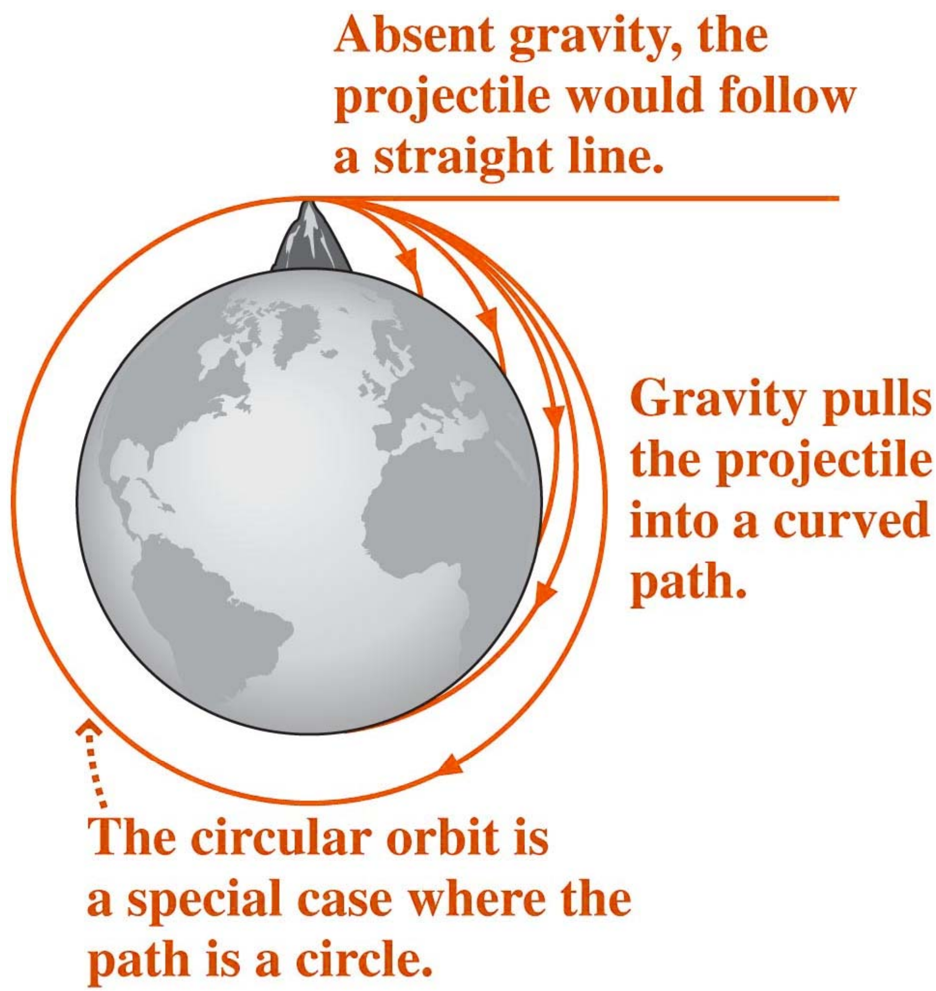
B. halved.

C. quadrupled.

D. quartered.

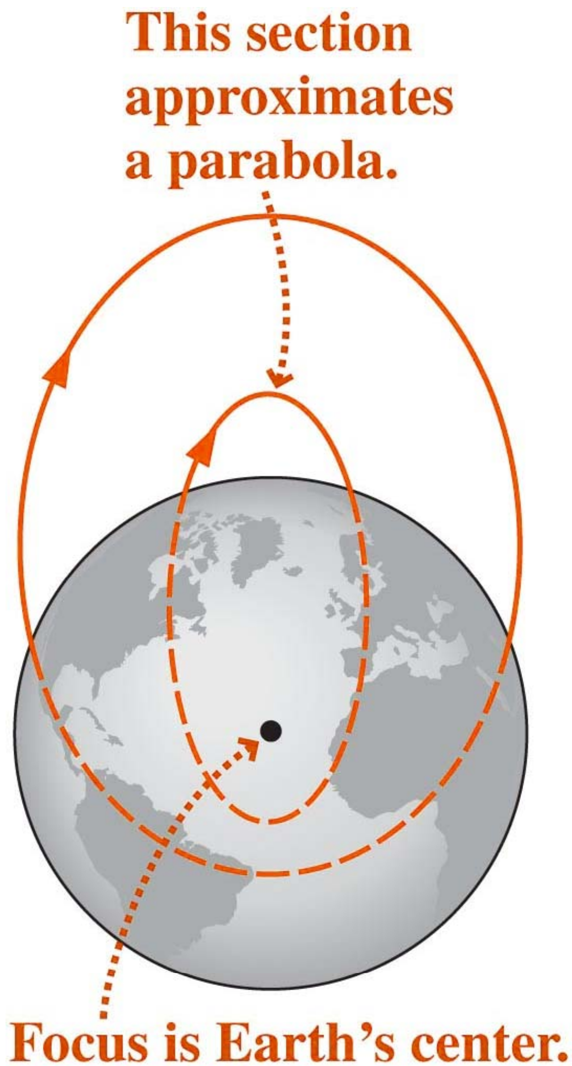
Orbits

- Newton explained orbits using universal gravitation and his laws of motion:
  - Bound orbits are generally elliptical.
  - In the special case of a circular orbit, the orbiting object "falls" around a gravitating mass, always accelerating toward its center with the magnitude of its acceleration remaining constant.
  - Unbound orbits are hyperbolic or (borderline case) parabolic.



Projectile motion and orbits

- The “parabolic” trajectories of projectiles near Earth’s surface are actually sections of elliptical orbits that intersect Earth.
- The trajectories are parabolic only in the approximation that we can neglect Earth’s curvature and the variation in gravity with distance from Earth’s center.



Circular orbits

- In a circular orbit, gravity provides the force of magnitude mv^2/r needed to keep an object of mass m in its circular path about a much more massive object of mass M.
- Therefore

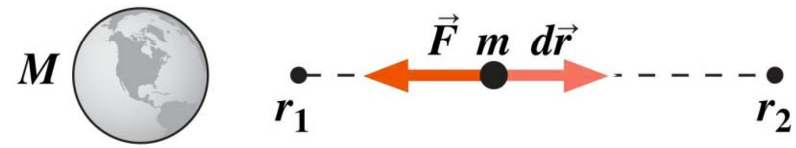
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

- This gives orbital speed $v = \sqrt{GM/r}$

  and orbital period $T^2 = \dfrac{4\pi^2 r^3}{GM}$.
  - For satellites in low-Earth orbit, the period is about 90 minutes.

Gravitational potential energy

- Because the gravitational force changes with distance, it's necessary to integrate to calculate potential energy changes over large distances. This integration gives

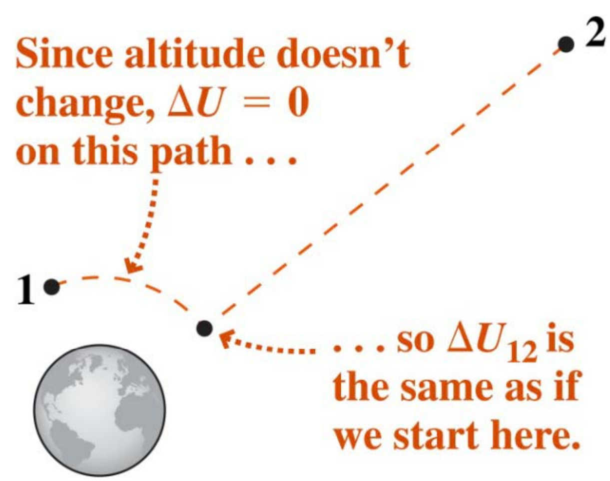


$$\Delta U_{12} = \int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr = GMm\int_{r_1}^{r_2} r^{-2}\,dr = GMm\left.\frac{r^{-1}}{-1}\right|_{r_1}^{r_2} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- This result holds regardless of whether the two points are on the same radial line.

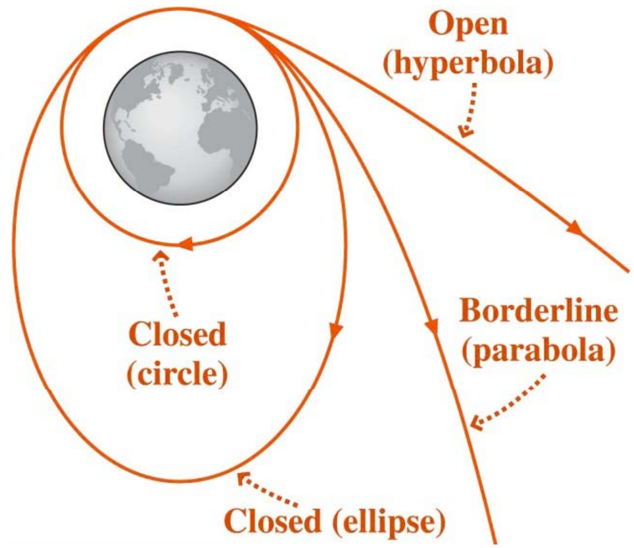
- It's convenient to take the zero of gravitational potential energy at infinity. Then the gravitational potential energy becomes

$$U(r) = -\frac{GMm}{r}$$

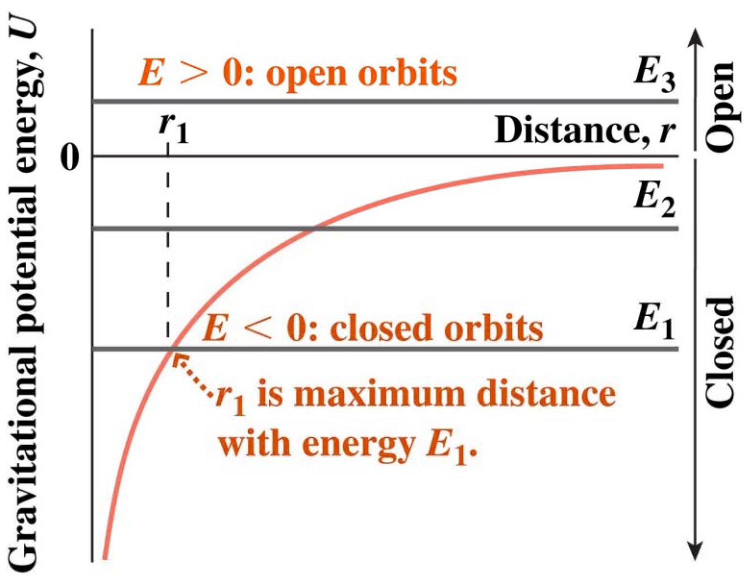


Energy and orbits

- The total energy E—the sum of kinetic energy K and potential energy U—determines the type of orbit an object follows:



  - For $E < 0$, the object is in a bound, elliptical orbit.
    - Special cases include circular orbits and the straight-line paths of falling objects.
  - For $E > 0$ the orbit is unbound and hyperbolic.
  - The borderline case $E = 0$ gives a parabolic orbit.



Escape speed

- An object with total energy *E* less than zero is in a bound orbit and can't escape from the gravitating center.
- With energy *E* greater than zero, the object is in an unbound orbit and can escape to infinitely far from the gravitating center.
- The minimum speed required to escape is given by

$$0 = K + U = \tfrac{1}{2}mv^2 - \frac{GMm}{r}$$

- Solving for *v* gives the escape speed:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

  - Escape speed from Earth's surface is about 11 km/s.

Energy in circular orbits

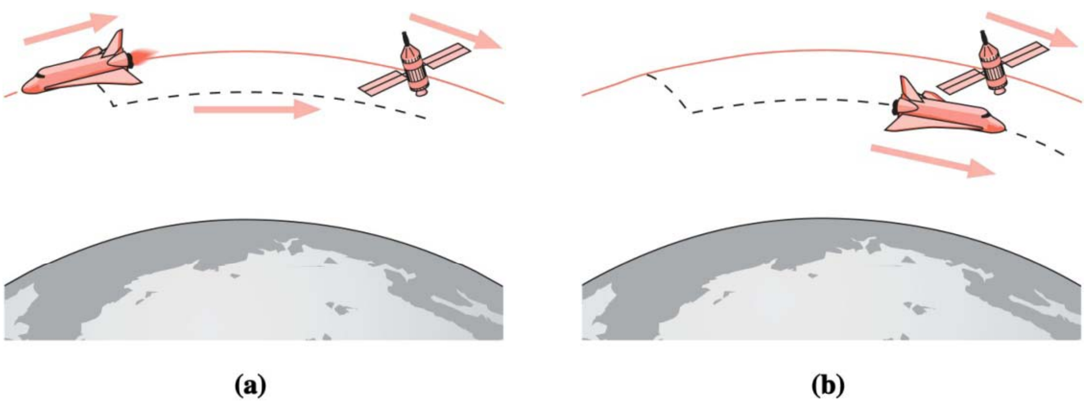
- In the special case of a circular orbit, kinetic energy and potential energy are precisely related:

$$U = -2K$$

- Thus in a circular orbit the total energy is

$$E = K + U = -K = \tfrac{1}{2}U = -\frac{GMm}{2r}$$

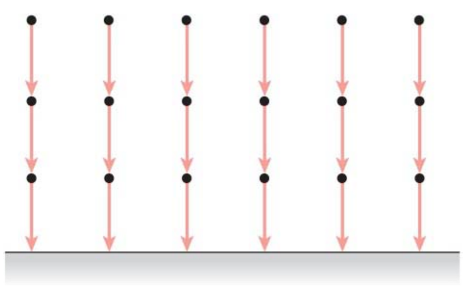
  - This negative energy shows that the orbit is bound.
  - The lower the orbit, the lower the total energy—but the faster the orbital speed.
    - This means an orbiting spacecraft needs to lose energy to gain speed.



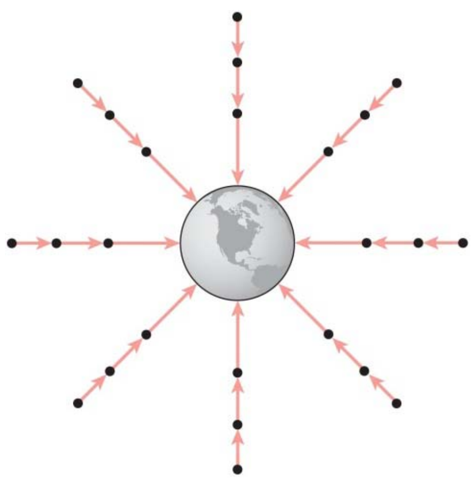
The gravitational field

- It's convenient to describe gravitation not in terms of "action at a distance" but rather in terms of a **gravitational field** that results from the presence of mass and that exists at all points in space.
  - A massive object creates a gravitational field in its vicinity, and other objects respond to the field *at their immediate locations*.
  - The gravitational field can be visualized with a set of vectors giving its strength (in N/kg; equivalently, m/s^2) and its direction.

Near Earth's surface:



On a larger scale:



Summary

- The **Law of Universal Gravitation** states that any two masses m_1 and m_2 attract with a force F that is proportional to the product of their distances and inversely proportional to the distance r between them:

$$F = \frac{Gm_1 m_2}{r^2}$$

- Motion under gravity includes
  - Bound elliptical and circular orbits when the orbiting object's total energy is less than zero
  - Open parabolic and hyperbolic orbits when the total energy is zero or greater
- The gravitational field describes the force of gravity in terms of a field at all points in space; an object then responds to the field in its immediate vicinity.