Physics 1501

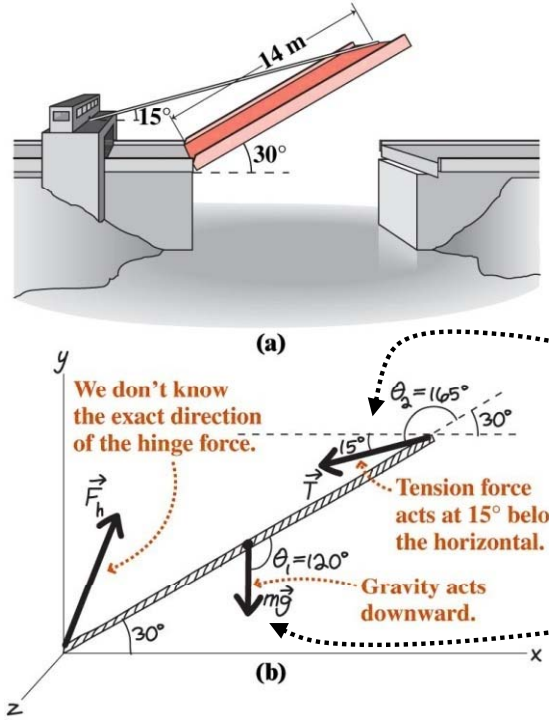
Fall 2008

Mechanics, Thermodynamics, Waves, Fluids

Lecture 25: Static Equilibrium II

Recap: Conditions for static equilibrium

- A system in static equilibrium undergoes neither rotational nor linear acceleration.
  - If it's at rest, it remains at rest.
- The conditions for static equilibrium are
  - No net force: $\sum \vec{F}_i = \vec{0}$
  - No net torque: $\sum \vec{\tau}_i = \sum \vec{r}_i \times \vec{F}_i = \vec{0}$
    - Torques can be evaluated about any convenient point:



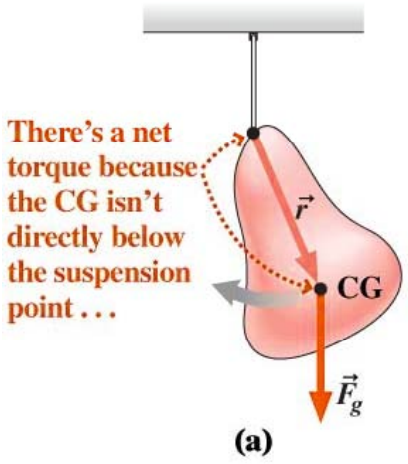
A drawbridge of length L and the forces acting on it. Choosing the pivot point at the lower left leaves two forces that contribute torques:

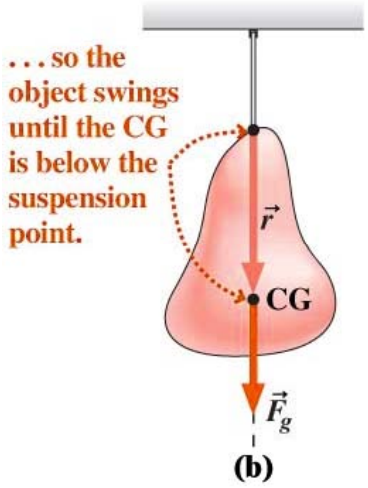
Torque due to tension $\vec{T}$ is $LT \sin\theta_2$.

Torque due to weight $m\vec{g}$ is $-\frac{L}{2} mg \sin\theta_1$.

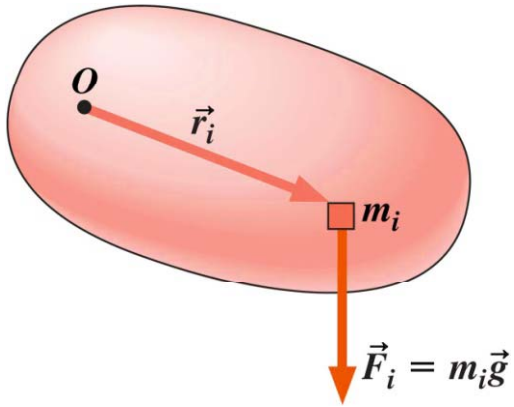
Recap: Center of gravity

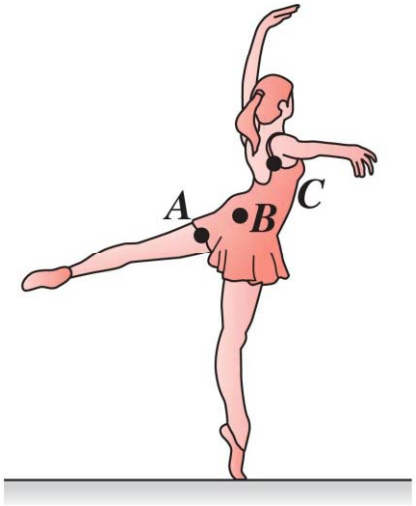
- The gravitational forces acting on all parts of an object exert a torque on the object.
  - These forces act like a single force, equal to the object's weight, acting at a point called the **center of gravity.**
  - In a uniform gravitational field, the center of gravity coincides with the center of mass.





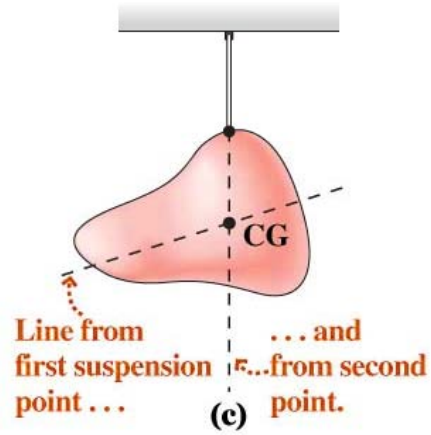
Gravitational force due to a single mass element produces a torque about *O*:





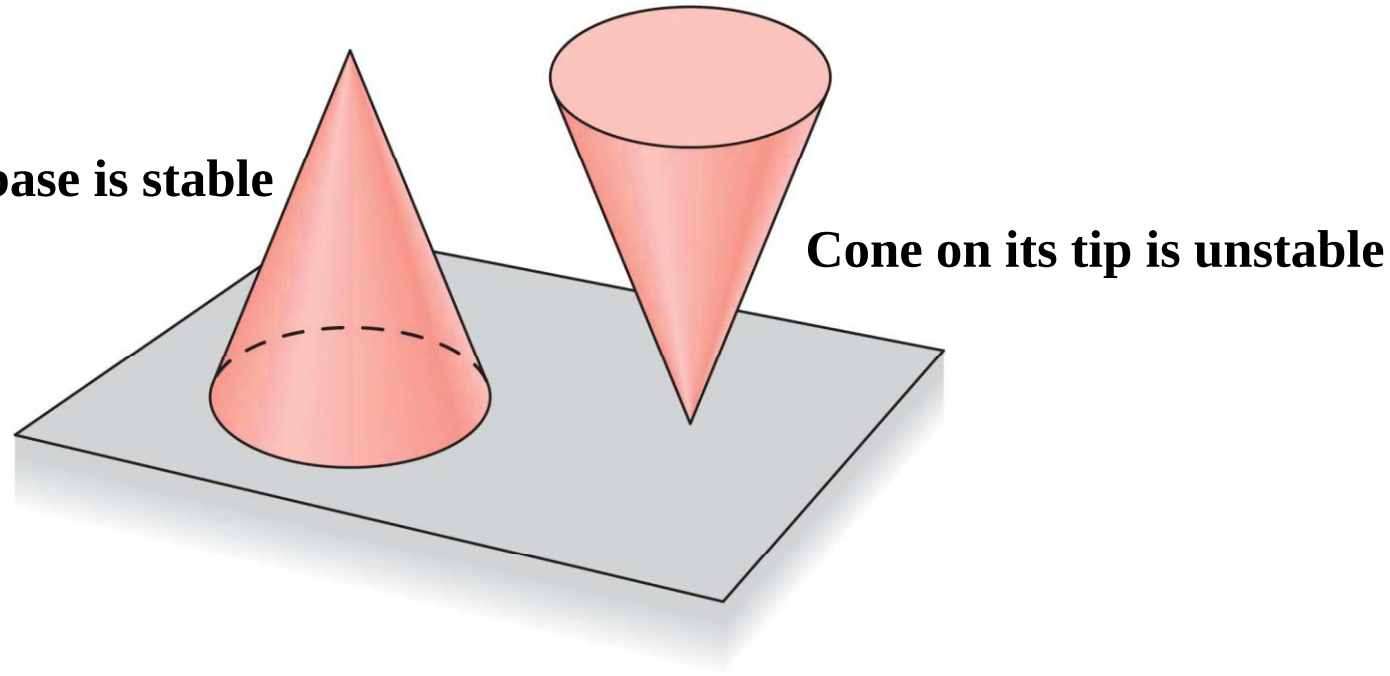
The dancer is in static equilibrium. Which point is her center of gravity?

Finding the center of gravity

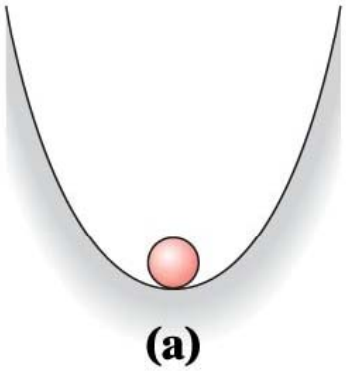


Stability

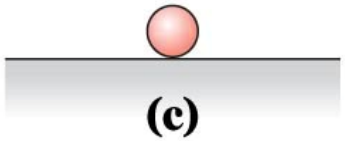
- An equilibrium is stable if a slight disturbance from equilibrium results in forces and/or torques that tend to restore the equilibrium.
- An equilibrium is unstable if a slight disturbance causes the system to move away from the original equilibrium.



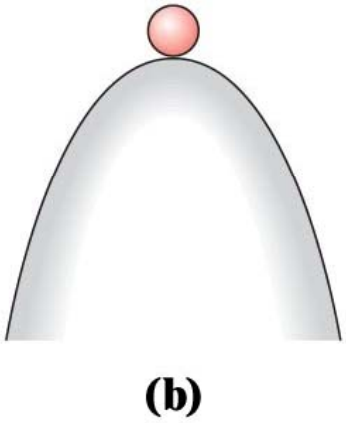
Kinds of stability



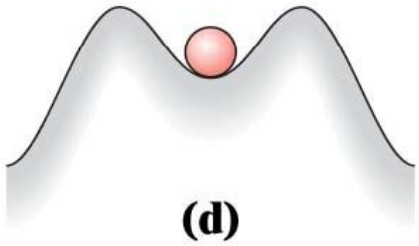
Stable equilibrium: disturbed ball will return to equilibrium



Neutrally stable equilibrium



Unstable equilibrium: disturbed ball will leave original equilibrium



Metastable or conditionally stable equilibrium: ball returns for small disturbances, but not for large ones

question

Which of the labeled points in the figure is in metastable equilibrium?

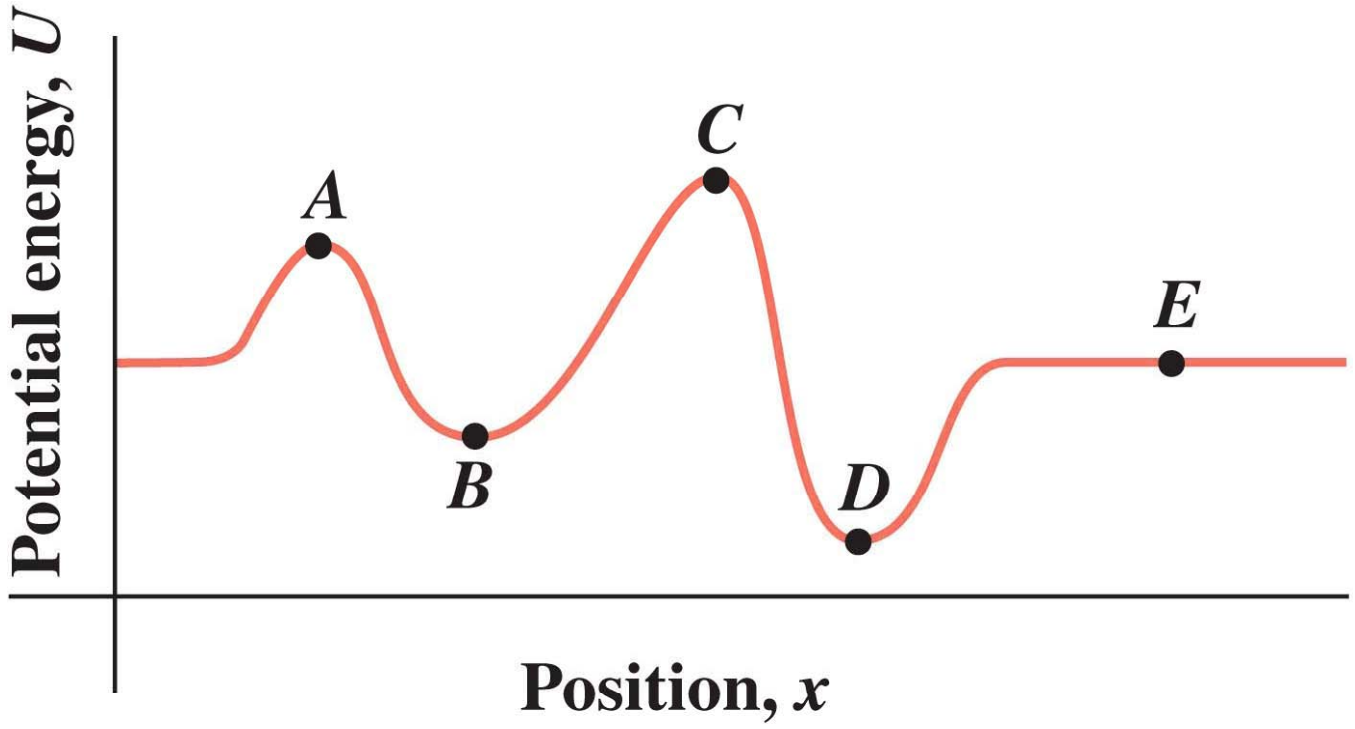
A. Point *A*

B. Point *B*

C. Point *C*

D. Point *D*

E. Point *E*



Conditions for equilibrium and stability

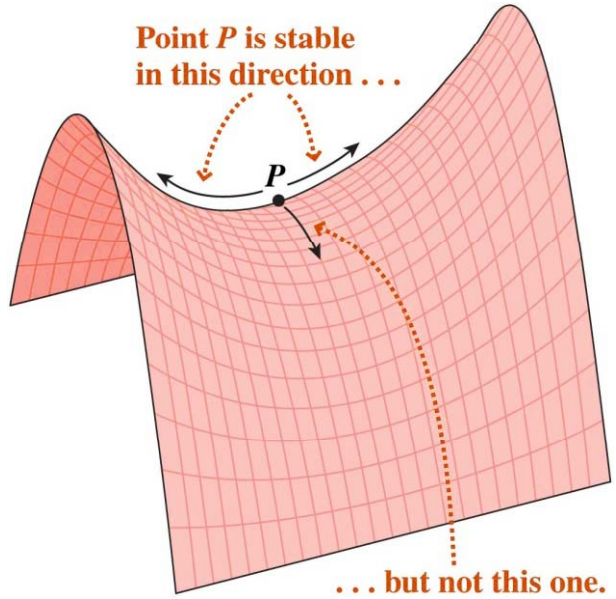
- To be in equilibrium, there must be zero net force on an object.
  - Therefore the object must be at a maximum or minimum of its potential energy curve:

$$\frac{dU}{dx} = 0 \quad \text{(condition for equilibrium)}$$

- For stable equilibrium, the object must be at a minimum:

$$\frac{d^2U}{dx^2} > 0 \quad \text{(stable equilibrium)}$$

- In two and three dimensions, an object can be stable in one direction but not another:



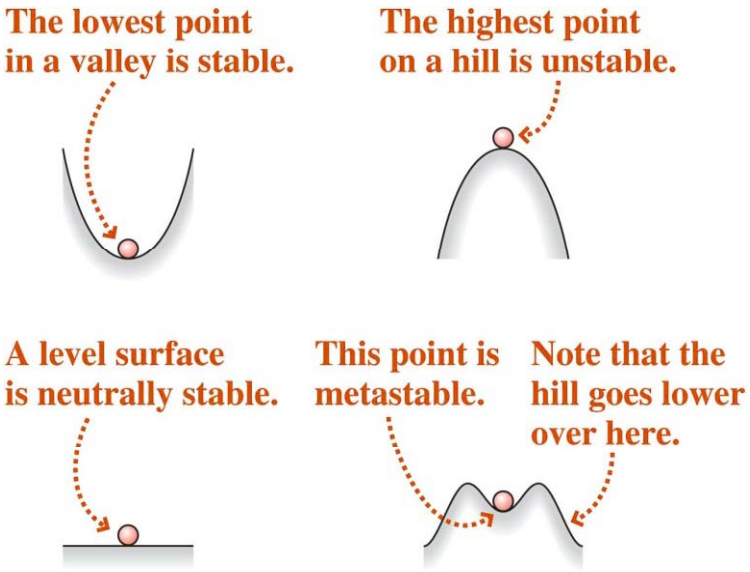
Summary

- Static equilibrium requires zero net force and zero net torque on a system:

$$\sum \vec{F}_i = \vec{0}$$

$$\sum \vec{\tau}_i = \sum \vec{r}_i \times \vec{F}_i = \vec{0}$$

- **Equilibria can be stable, unstable, neutrally stable, or metastable:**



A crane in static equilibrium