Physics 1501

Fall 2008

Mechanics, Thermodynamics, Waves, Fluids

Lecture 22: Angular Momentum

Recap : rotational and linear dynamics

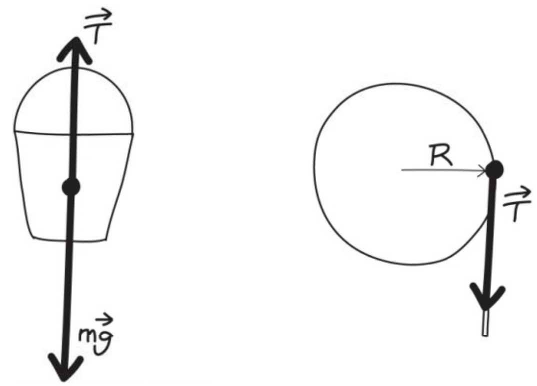
- In problems involving both linear and rotational motion:
  - IDENTIFY the objects and forces or torques acting.
  - DEVELOP your solution with drawings and by writing Newton's law and its rotational analog. Note physical connections between the objects.
  - EVALUATE to find the solution.
  - ASSESS to be sure your answer makes sense.

A bucket of mass *m* drops into a well, its rope unrolling from a cylinder of mass *M* and radius *R*

What's its acceleration?

Freebody diagrams for bucket and cylinder

Rope tension $\vec{T}$ provides the connection



Newton's law, bucket:
$F_{net} = mg - T = ma$

Rotational analogy of Newton's law, cylinder:
$RT = Ia/R$

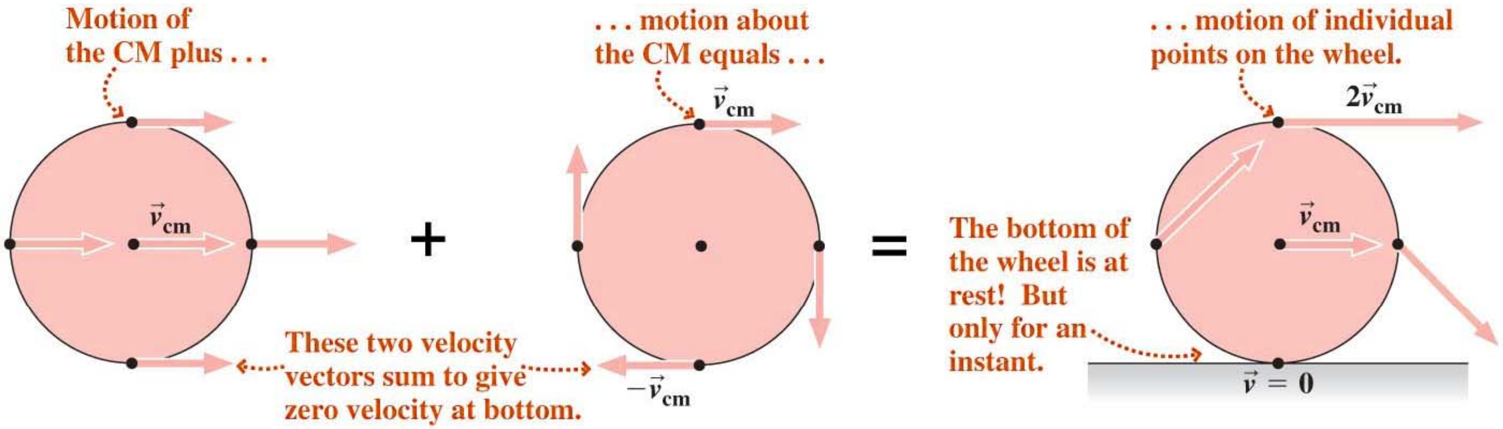
Here $I = \frac{1}{2}MR^2$

Solve the two equations to get

$$a = \frac{mg}{m + \frac{1}{2}M}$$

Recap: rolling motion

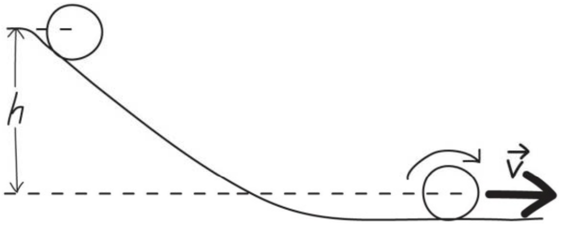
- Rolling motion combines translational (linear) motion and rotational motion.
  - The rolling object's center of mass undergoes translational motion.
  - The object itself rotates about the center of mass.
  - In true rolling motion, the object moves without slipping and its point of contact with the ground is instantaneously at rest.
  - Then the rotational speed ω and linear speed v are related by $v = \omega R$, where R is the object's radius.



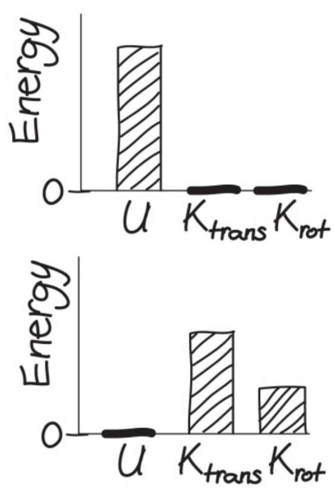
Recap: rotational energy

- A rotating object has kinetic energy $K_{\text{rot}} = \frac{1}{2} I\omega^2$ associated with its rotational motion alone.
  - It may also have translational kinetic energy: $K_{\text{trans}} = \frac{1}{2} Mv^2$.
- In problems involving energy conservation with rotating objects, both forms of kinetic energy must be considered.
  - For rolling objects, the two are related:
    - The relation depends on the rotational inertia.

A solid ball rolls down a hill.
How fast is it moving at the bottom?



Energy bar



Equation for energy conservation

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{7}{10}Mv^2$$

Solution

$$v = \sqrt{\frac{10}{7}gh}$$

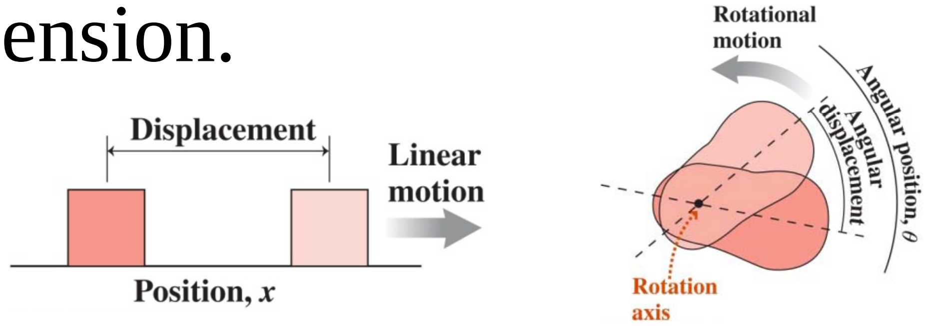
question

A hollow ball and a solid ball roll without slipping down an inclined plane. Which ball reaches the bottom of the incline first?

A. The solid ball reaches the bottom first.
B. The hollow ball reaches the bottom first.
C. Both balls reach the bottom at the same time.
D. We can’t determine this without information about the mass.

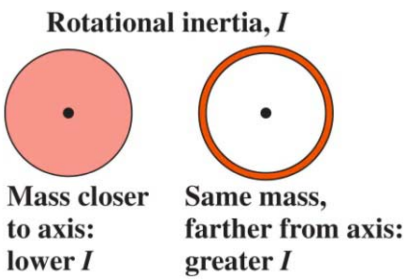
Summary

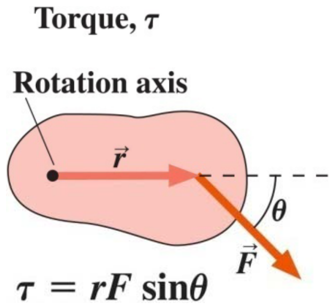
- Rotational motion in one dimension is exactly analogous to linear motion in one dimension.
  - Linear and angular motion:



  - Analogies between rotational and linear quantities:

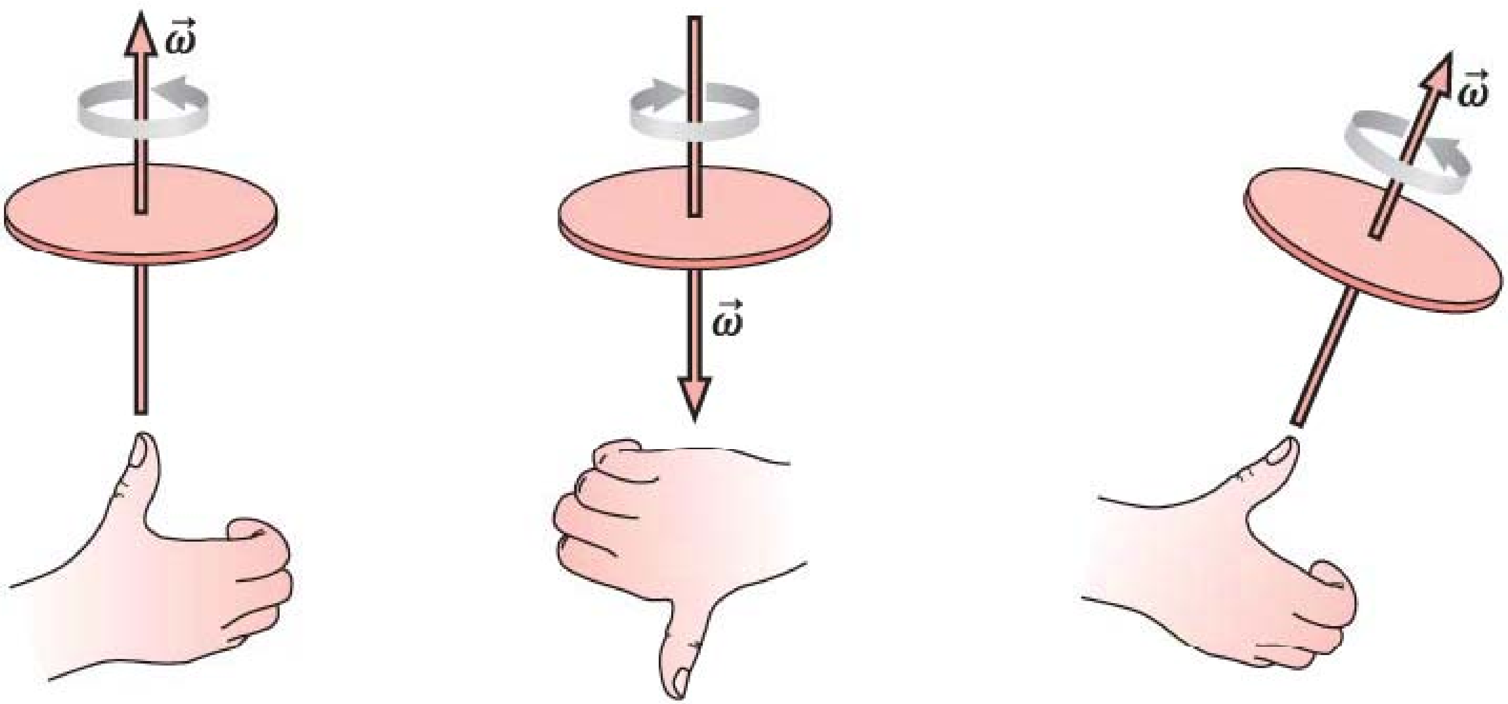
Linear Quantity or Equation	Angular Quantity or Equation	Relation Between Linear and Angular Quantities
Position x	Angular position θ	
Speed $v = dx/dt$	Angular speed $\omega = d\theta/dt$	$v = \omega r$
Acceleration a	Angular acceleration α	$a_t = \alpha r$
Mass m	Rotational inertia I	$I = \int r^2\, dm$
Force F	Torque τ	$\tau = rF \sin\theta$
Kinetic energy $K_{\text{trans}} = \frac{1}{2}mv^2$	Kinetic energy $K_{\text{rot}} = \frac{1}{2}I\omega^2$	
Newton's second law (constant mass or rotational inertia):		
$F = ma$	$\tau = I\alpha$	





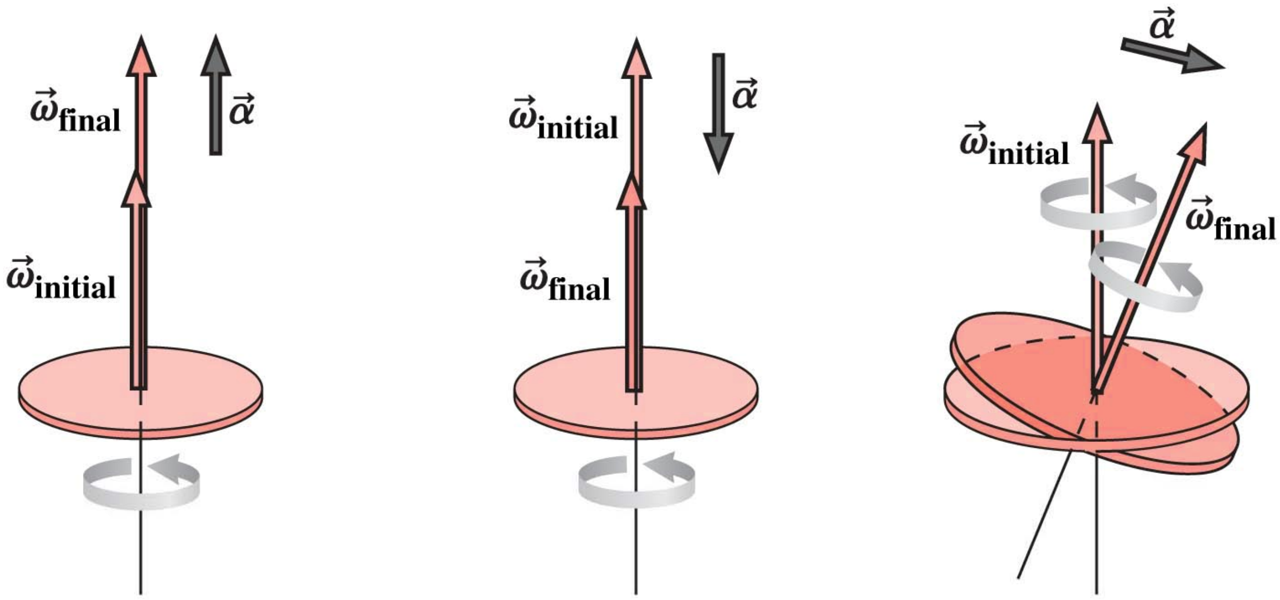
Direction of the angular velocity vector

- The direction of angular velocity is given by the **right-hand rule.**
  - Curl the fingers of your right hand in the direction of rotation, and your thumb points in the direction of the angular velocity vector $\dot{\omega}$.



Direction of the angular acceleration

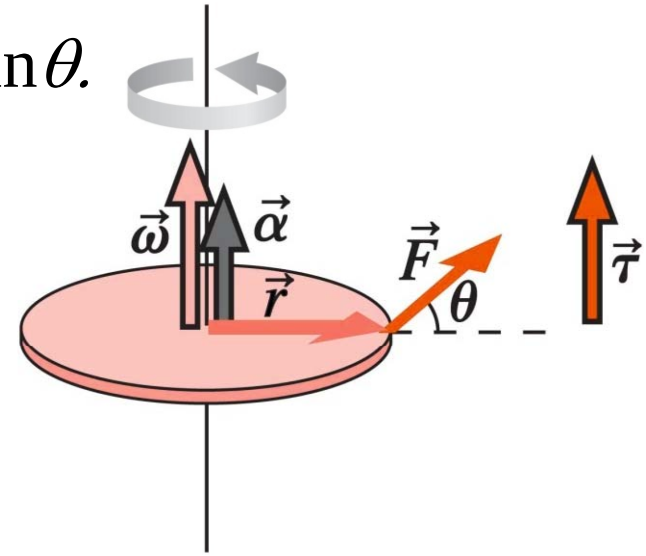
- Angular acceleration points in the direction of the change $\Delta\vec{\omega}$ in the angular velocity.
  - The change can be in the same direction as the angular velocity, increasing the angular speed.
  - The change can be opposite the angular velocity, decreasing the angular speed.
  - Or it can be in an arbitrary direction, changing the direction and speed as well.

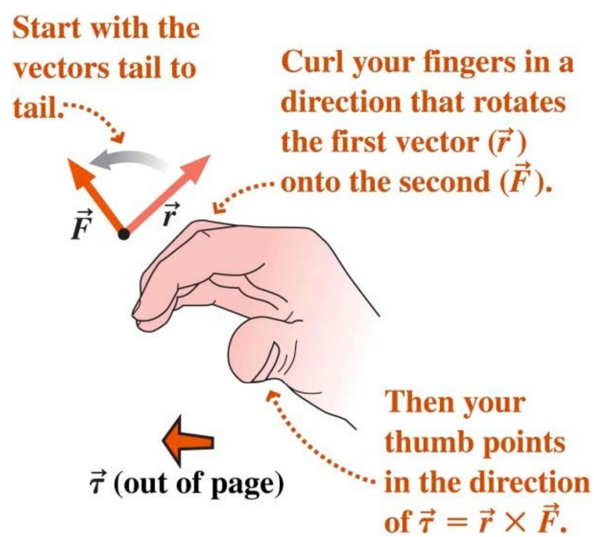


Direction of the torque vector

- The torque vector is perpendicular to both the force vector and the displacement vector from the rotation axis to the force application point.
  - The magnitude of the torque is $\tau = rF\sin\theta$.
  - Of the two possible directions perpendicular to $\vec{r}$ and $\vec{F}$, the correct direction is given by the right-hand rule.
  - Torque is compactly expressed using the vector cross product:

$$\vec{\tau} = \vec{r} \times \vec{F}$$





question

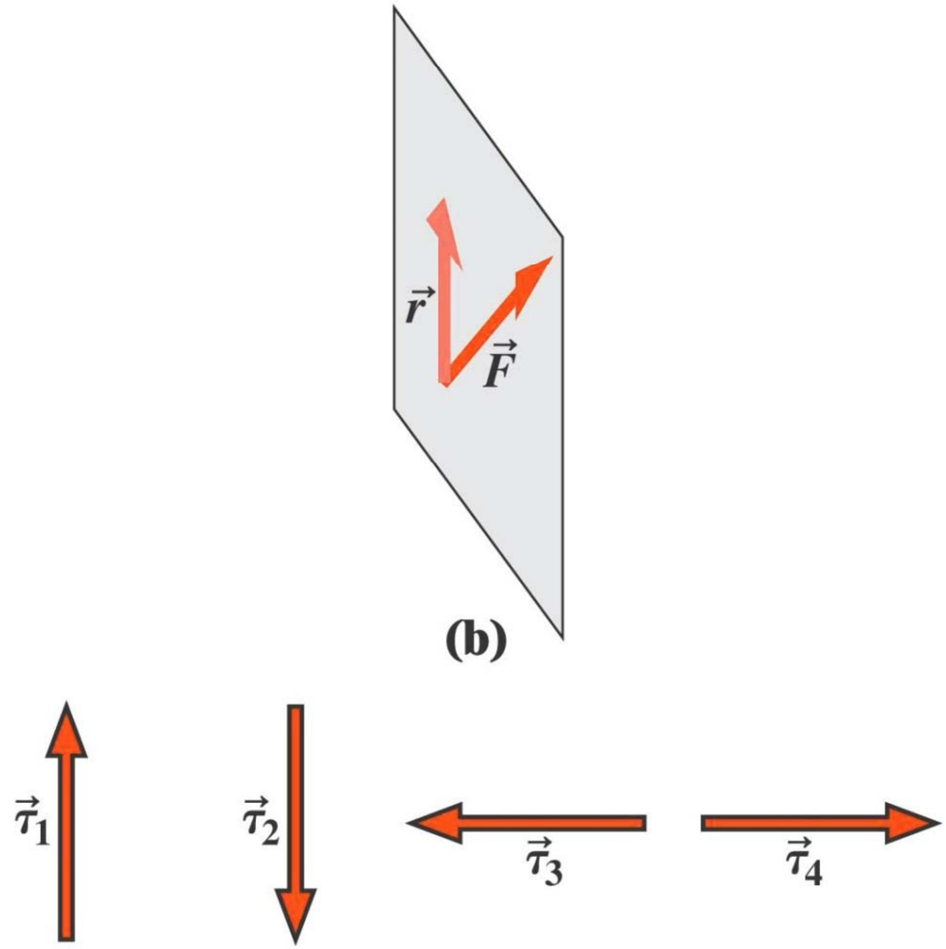
The figure shows a pair of force and radius vectors and four torque vectors. Which of the numbered torque vectors goes with the force and radius vectors?

A. **Torque vector $\boldsymbol{\tau}_1$**

B. **Torque vector $\boldsymbol{\tau}_2$**

C. **Torque vector $\boldsymbol{\tau}_3$**

D. **Torque vector $\boldsymbol{\tau}_4$**



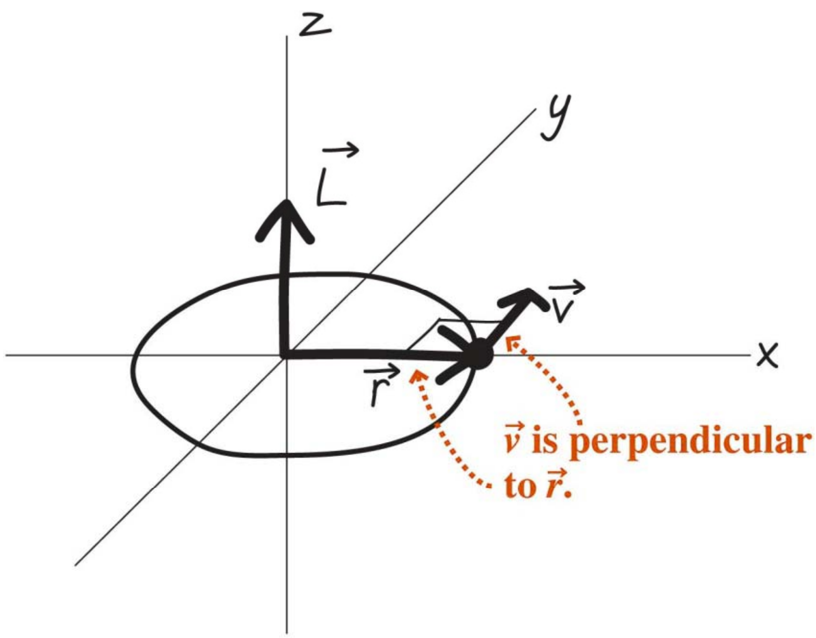
Angular momentum

- For a single particle, angular momentum $\vec{L}$ is a vector given by the cross product of the displacement vector from the rotation axis with the linear momentum of the particle:

$$\vec{L} = \vec{r} \times \vec{p}$$

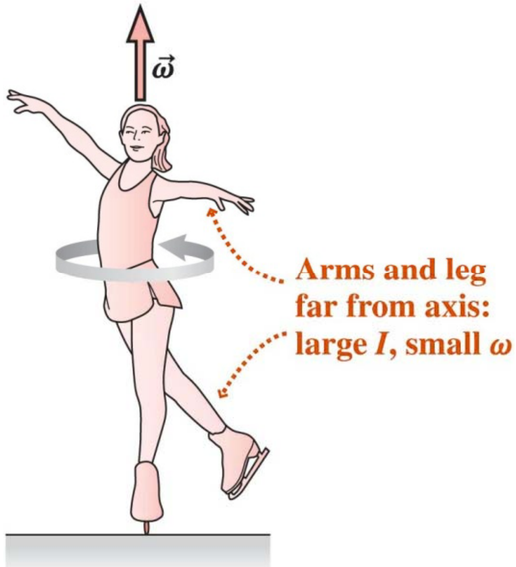
  - For the case of a particle in a circular path, $L = mvr$, and $\vec{L}$ is upward, perpendicular to the circle.
  - For sufficiently symmetric objects, $\vec{L}$ is the product of rotational inertia and angular velocity:

$$\vec{L} = I\vec{\omega}$$

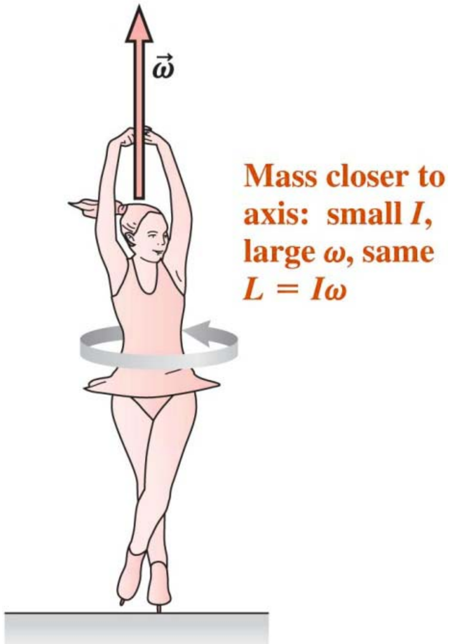


Newton's law and angular momentum

- In terms of angular momentum, the rotational analog of Newton's second law is $$\vec{\tau} = \frac{d\vec{L}}{dt}$$
  - Therefore a system's angular momentum changes only if there's a nonzero net torque acting on the system.
  - If the net torque is zero, then angular momentum is conserved.
    - Changes in rotational inertia then result in changes in angular speed:

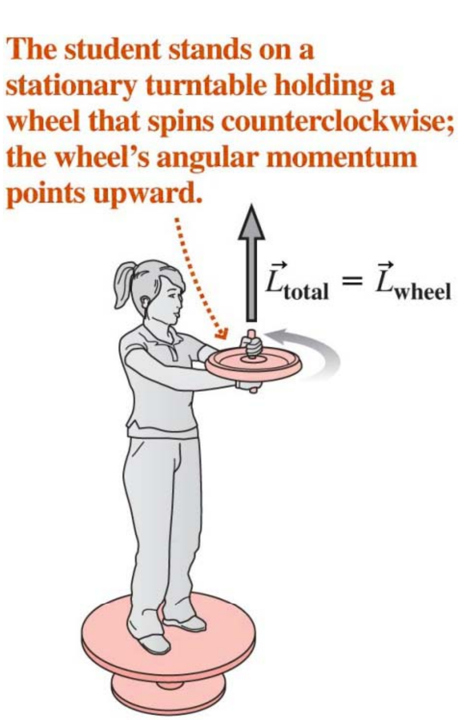


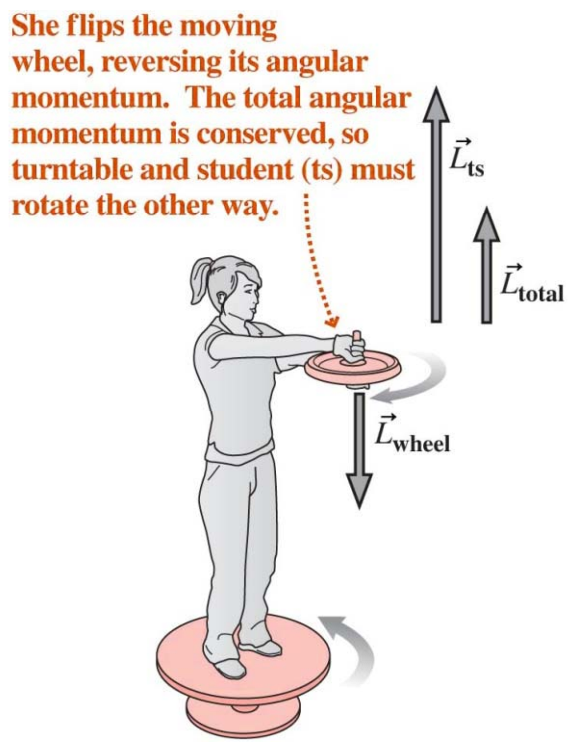
The skater's angular momentum is conserved, so her angular speed increases when she reduces her rotational inertia.



Conservation of angular momentum

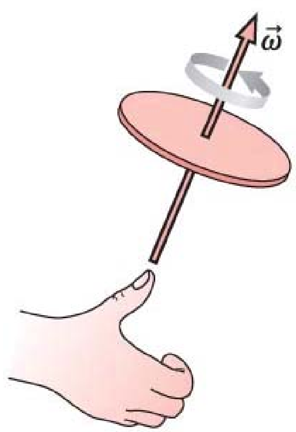
- The spinning wheel initially contains all the system's angular momentum.
- When the student turns the wheel upside down, she changes the direction of its angular momentum vector.
- Student and turntable rotate the other way to keep the total angular momentum unchanged.

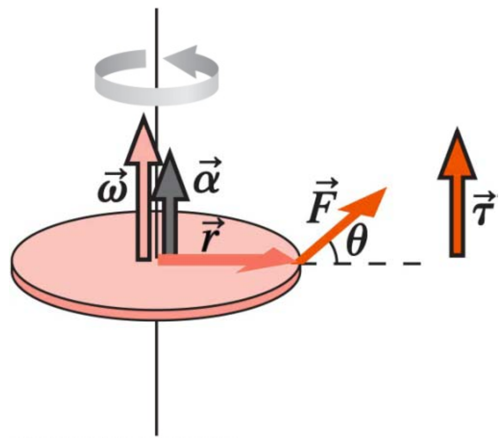


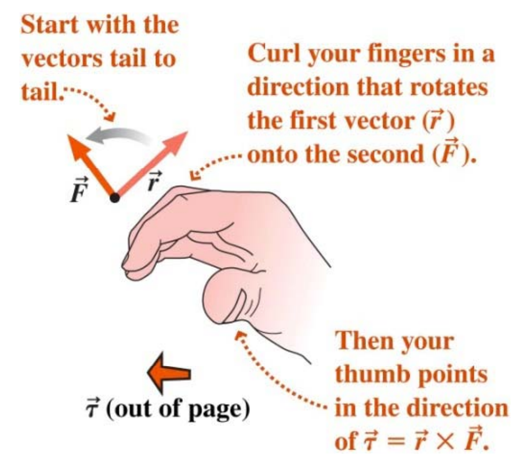


Summary

- Angular quantities are vectors whose direction is generally associated with the direction of the rotation axis.
  - Specifically, direction is given by the right-hand rule.
  - The vector cross product provides a compact representation for torque and angular momentum.







  - Angular momentum is the rotational analog of linear momentum: $\vec{L} = \vec{r} \times \vec{p}$; with symmetry, $\vec{L} = I\vec{\omega}$.
  - In the absence of a net external torque, a system's angular momentum is conserved.