Physics 1501

Fall 2008

Mechanics, Thermodynamics, Waves, Fluids

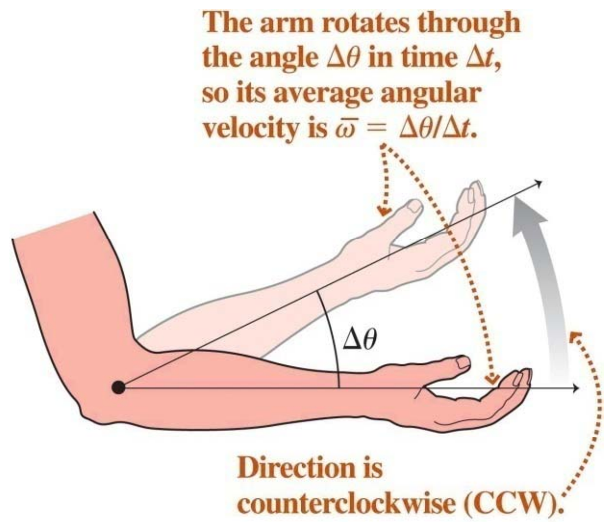
Lecture 21: Rotational Motion II

Recap: angular velocity

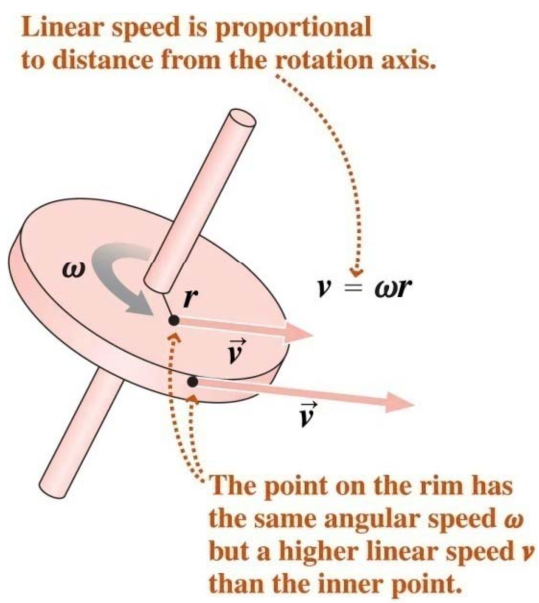
- Concept of a **rigid body**
- Angular velocity ω is the rate of change of angular position.

Average: $\bar{\omega} = \frac{\Delta\theta}{\Delta t}$

Instantaneous: $\omega = \frac{d\theta}{dt}$



- Angular and linear velocity
  - The linear speed of a point on a rotating body is proportional to its distance from the rotation axis: $v = \omega r$



Recap: angular acceleration

- Angular acceleration α is the rate of change of angular velocity.

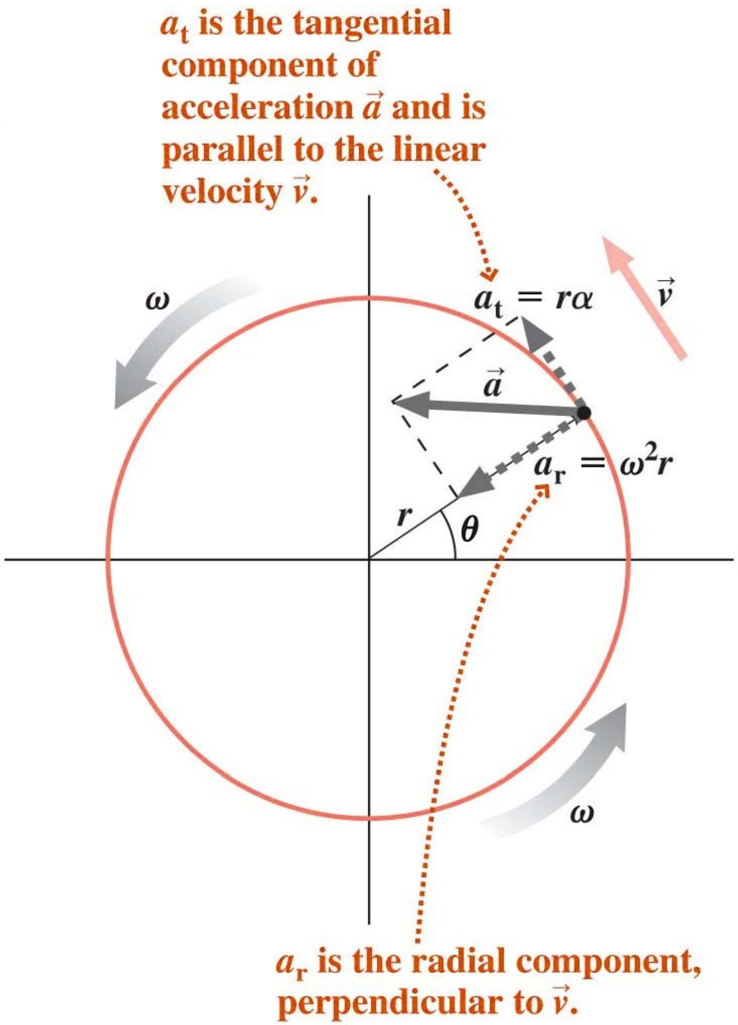
Average: $\bar{\alpha} = \dfrac{\Delta\omega}{\Delta t}$ Instantaneous: $\alpha = \dfrac{d\omega}{dt}$

- Angular and tangential acceleration
  - The linear acceleration of a point on a rotating body is proportional to its distance from the rotation axis:

$$a_{t} = r\,\alpha$$

  - A point on a rotating object also has radial acceleration:

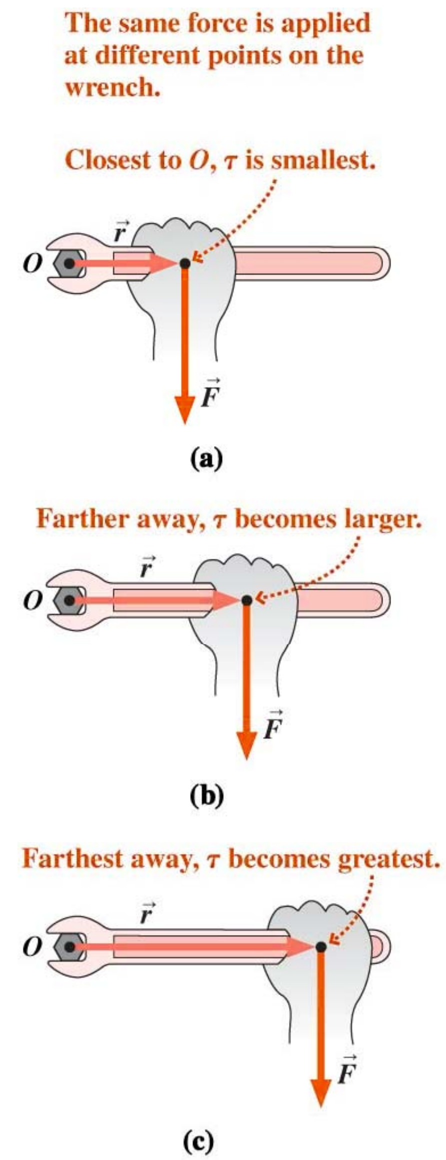
$$a_{r} = \frac{v^2}{r} = \omega^2 r$$



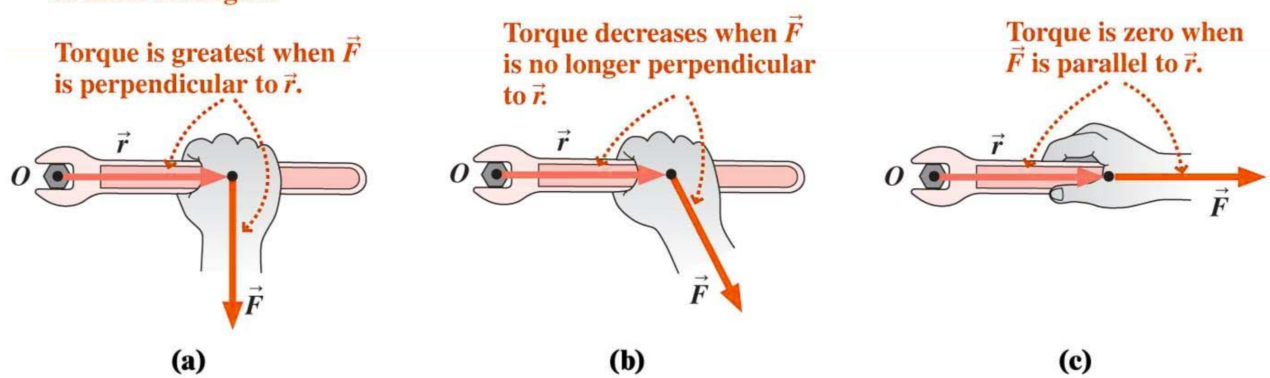
Recap: Torque

- Torque τ is the rotational analog of force, and results from the application of one or more forces.
  - Torque is relative to a chosen rotation axis.
  - Torque depends on
    - The distance from the rotation axis to the force application point.
    - The magnitude of the force $\vec{F}$.
    - The orientation of the force relative to the displacement $\vec{r}$ from axis to force application point:

$$\tau = rF\sin\theta$$



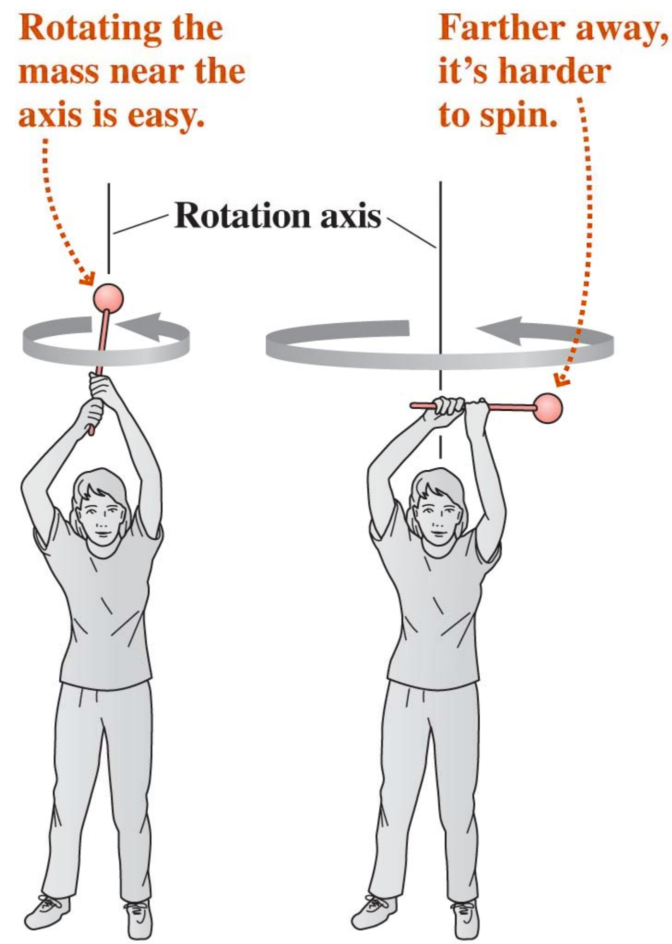
The same force is applied at different angles.



Recap: rotational analog of Newton's law

- Rotational inertia *I* is the rotational analog of mass.
  - Rotational inertia depends on mass and its distance from the rotation axis.
- Rotational acceleration, torque, and rotational inertia combine to give the rotational analog of Newton's second law:

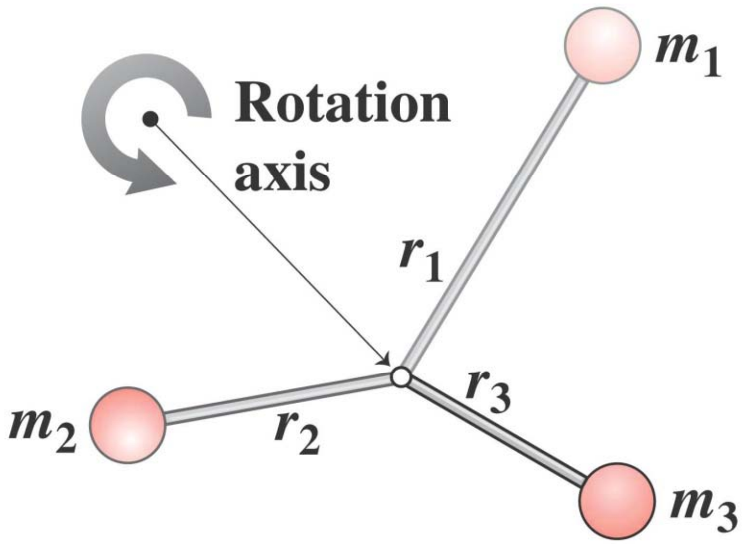
$$\tau = I\,\alpha$$



Recap: finding rotational inertia

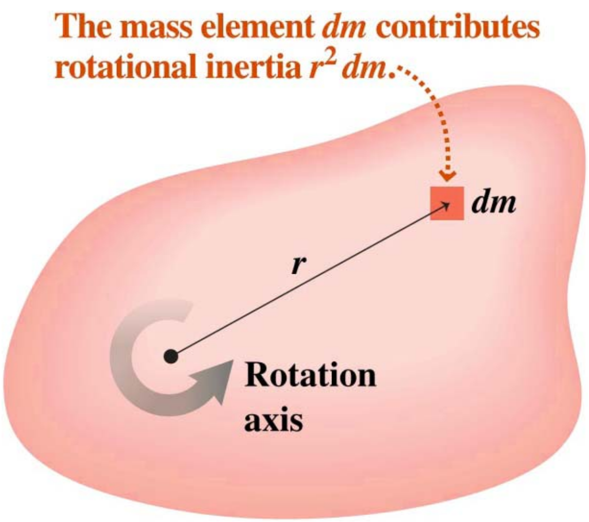
- For a single point mass m, rotational inertia is the product of mass with the square of the distance R from the rotation axis: $I = mR^2$.
- For a system of discrete masses, the rotational inertia is the sum of the rotational inertias of the individual masses:

$$I = \sum m_i r_i^2$$



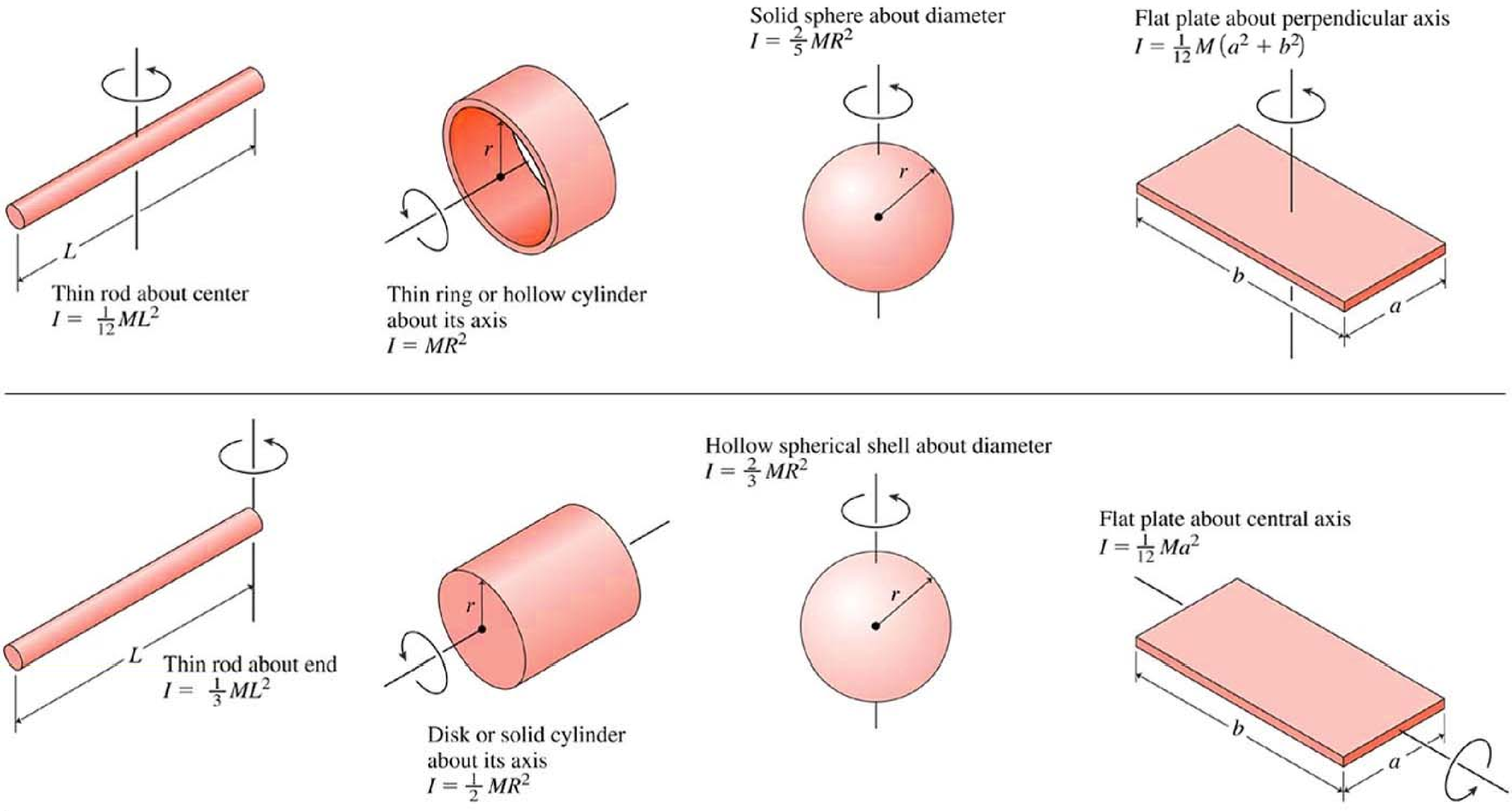
- For continuous matter, the rotational inertia is given by an integral over the distribution of matter:

$$I = \int r^2\, dm$$



- **Parallel axis theorem**

Rotational inertias of simple objects



Combining rotational and linear dynamics

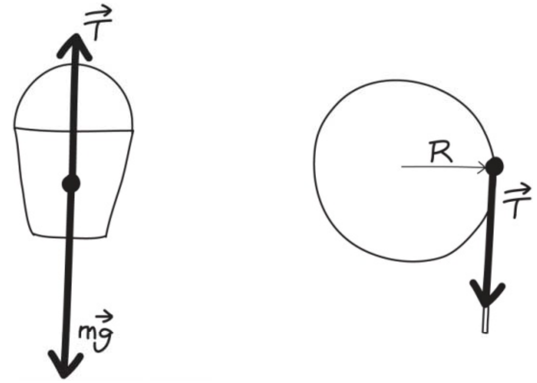
- In problems involving both linear and rotational motion:
  - IDENTIFY the objects and forces or torques acting.
  - DEVELOP your solution with drawings and by writing Newton's law and its rotational analog. Note physical connections between the objects.
  - EVALUATE to find the solution.
  - ASSESS to be sure your answer makes sense.

A bucket of mass *m* drops into a well, its rope unrolling from a cylinder of mass *M* and radius *R*

What's its acceleration?

Freebody diagrams for bucket and cylinder

Rope tension $\vec{T}$ provides the connection



Newton's law, bucket:
$F_{net} = mg - T = ma$

Rotational analogy of Newton's law, cylinder:
$RT = Ia/R$

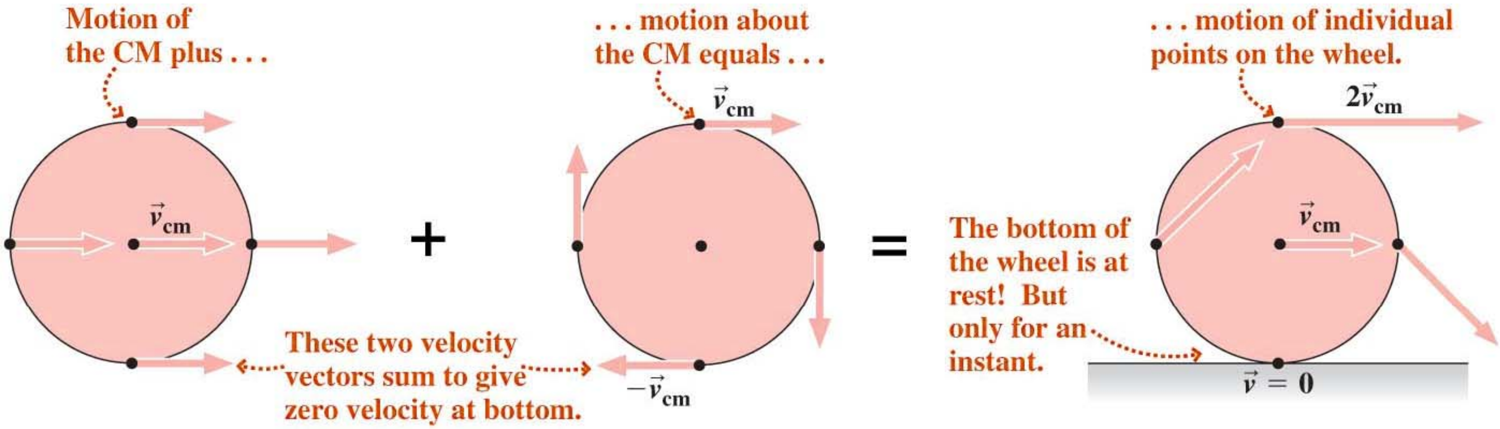
Here $I = \frac{1}{2}MR^2$

Solve the two equations to get

$$a = \frac{mg}{m + \frac{1}{2}M}$$

Rolling motion

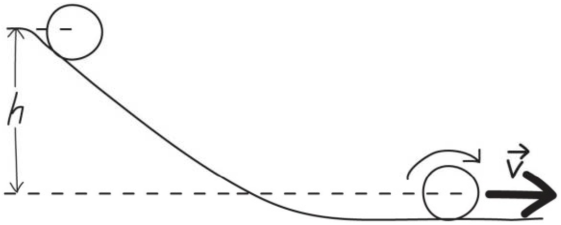
- Rolling motion combines translational (linear) motion and rotational motion.
  - The rolling object's center of mass undergoes translational motion.
  - The object itself rotates about the center of mass.
  - In true rolling motion, the object moves without slipping and its point of contact with the ground is instantaneously at rest.
  - Then the rotational speed ω and linear speed v are related by $v = \omega R$, where R is the object's radius.



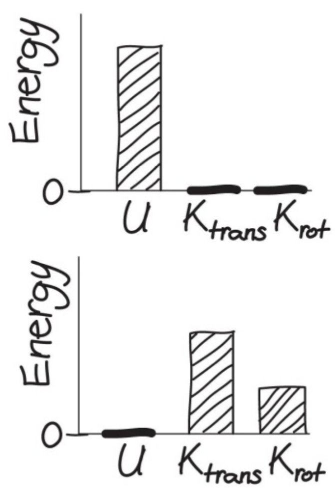
Rotational energy

- A rotating object has kinetic energy $K_{\text{rot}} = \frac{1}{2} I\omega^2$ associated with its rotational motion alone.
  - It may also have translational kinetic energy: $K_{\text{trans}} = \frac{1}{2} Mv^2$.
- In problems involving energy conservation with rotating objects, both forms of kinetic energy must be considered.
  - For rolling objects, the two are related:
    - The relation depends on the rotational inertia.

A solid ball rolls down a hill.
How fast is it moving at the bottom?



Energy bar



Equation for energy conservation

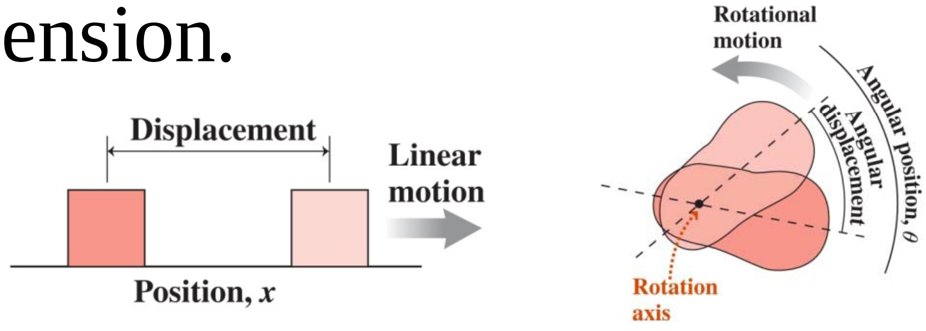
$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{7}{10}Mv^2$$

Solution

$$v = \sqrt{\frac{10}{7}gh}$$

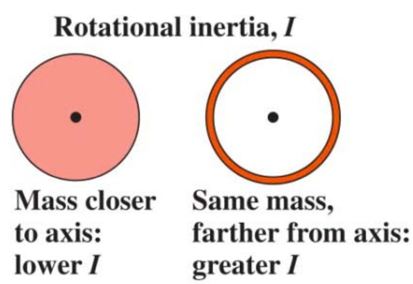
Summary

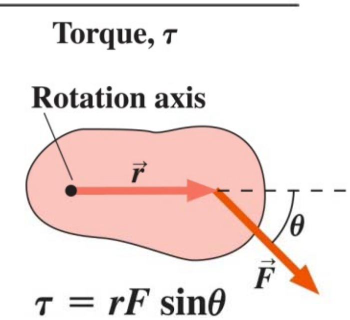
- Rotational motion in one dimension is exactly analogous to linear motion in one dimension.
  - Linear and angular motion:



  - Analogies between rotational and linear quantities:

Linear Quantity or Equation	Angular Quantity or Equation	Relation Between Linear and Angular Quantities
Position x	Angular position θ	
Speed $v = dx/dt$	Angular speed $\omega = d\theta/dt$	$v = \omega r$
Acceleration a	Angular acceleration α	$a_t = \alpha r$
Mass m	Rotational inertia I	$I = \int r^2\, dm$
Force F	Torque τ	$\tau = rF \sin\theta$
Kinetic energy $K_{\text{trans}} = \frac{1}{2}mv^2$	Kinetic energy $K_{\text{rot}} = \frac{1}{2}I\omega^2$	
Newton's second law (constant mass or rotational inertia):		
$F = ma$	$\tau = I\alpha$	





Clicker question

A hollow ball and a solid ball roll without slipping down an inclined plane. Which ball reaches the bottom of the incline first?

A. The solid ball reaches the bottom first.

B. The hollow ball reaches the bottom first.

C. Both balls reach the bottom at the same time.

D. We can't determine this without information about the mass.