Physics 1501

Fall 2008

Mechanics, Thermodynamics, Waves, Fluids

Lecture 20: Rotational Motion

Recap: center of mass, linear momentum

- A composite system behaves as though its mass is concentrated at the **center of mass:**

$$\vec{r}_{\text{cm}} = \frac{\sum m_i \vec{r}_i}{M} \quad \text{(discrete particles)} \qquad \vec{r}_{\text{cm}} = \frac{\int \vec{r}\, dm}{M} \quad \text{(continuous matter)}$$

- The center of mass obeys Newton's laws, so

$$\vec{F}_{\text{net external}} = M\vec{a}_{\text{cm}} \text{ or, equivalently, } \vec{F}_{\text{net external}} = \frac{d\vec{P}}{dt}$$

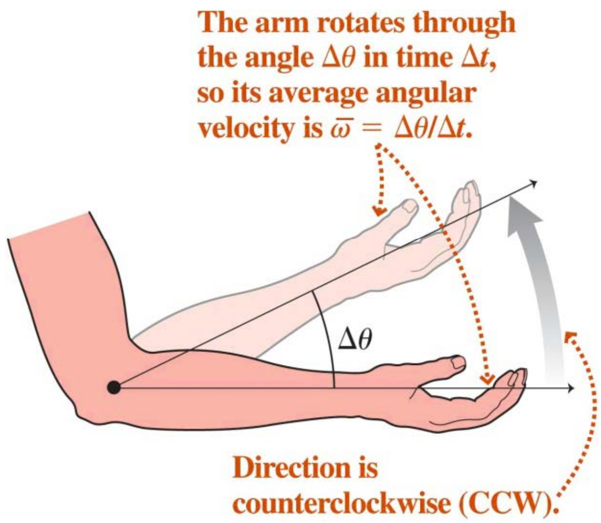
- In the absence of a net external force, a system's linear momentum is conserved, regardless of what happens internally to the system.
- Collisions are brief, intense interactions that conserve momentum.
  - Elastic collisions also conserve kinetic energy.
  - Totally inelastic collisions occur when colliding objects join to make a single composite object.

Angular velocity

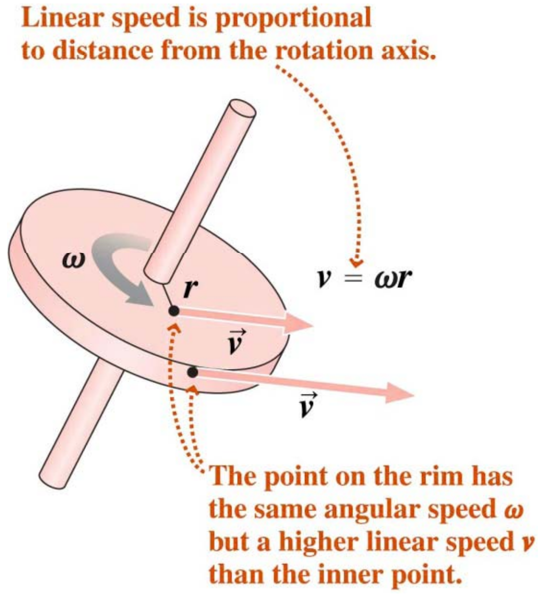
- Concept of a **rigid body**
- Angular velocity ω is the rate of change of angular position.

Average: $$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous: $$\omega = \frac{d\theta}{dt}$$



- Angular and linear velocity
  - The linear speed of a point on a rotating body is proportional to its distance from the rotation axis: $$v = \omega r$$



Angular acceleration

- Angular acceleration α is the rate of change of angular velocity.

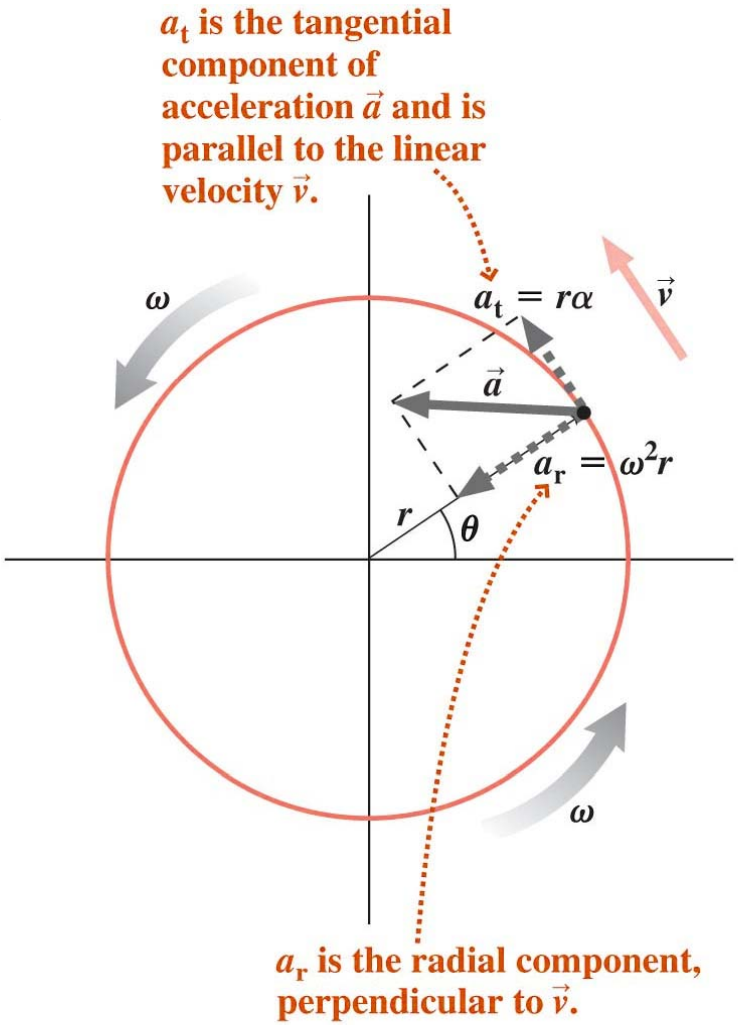
$$\text{Average: } \bar{\alpha} = \frac{\Delta\omega}{\Delta t} \qquad \text{Instantaneous: } \alpha = \frac{d\omega}{dt}$$

- Angular and tangential acceleration
  - The linear acceleration of a point on a rotating body is proportional to its distance from the rotation axis:

$$a_{\mathrm{t}} = r\,\alpha$$

  - A point on a rotating object also has radial acceleration:

$$a_{\mathrm{r}} = \frac{v^2}{r} = \omega^2 r$$



Constant angular acceleration

- Problems with constant angular acceleration are exactly analogous to similar problems involving linear motion in one dimension.
  - The same equations apply, with the substitutions

    $x \to \theta, \quad v \to \omega, \quad a \to \alpha$

TABLE 10.1 Angular and Linear Position, Velocity, and Acceleration

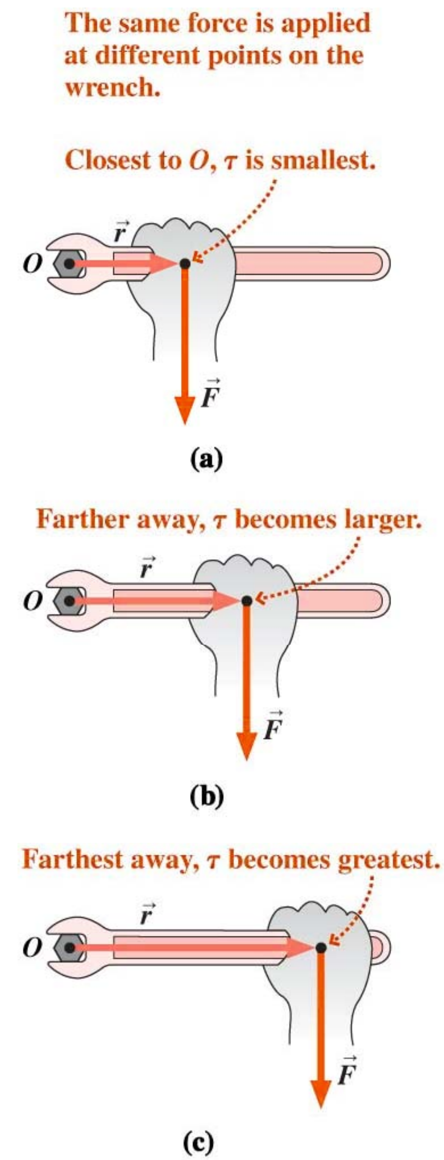
Linear Quantity	**Angular Quantity**
Position x	Angular position θ
Velocity $v = \dfrac{dx}{dt}$	Angular velocity $\omega = \dfrac{d\theta}{dt}$
Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$	Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

Equations for Constant Linear Acceleration		**Equations for Constant Angular Acceleration**	
$\overline{v} = \frac{1}{2}(v_0 + v)$	(2.8)	$\overline{\omega} = \frac{1}{2}(\omega_0 + \omega)$	(10.6)
$v = v_0 + at$	(2.7)	$\omega = \omega_0 + \alpha t$	(10.7)
$x = x_0 + v_0 t + \frac{1}{2}at^2$	(2.10)	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	(10.8)
$v^2 = v_0^2 + 2a(x - x_0)$	(2.11)	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10.9)

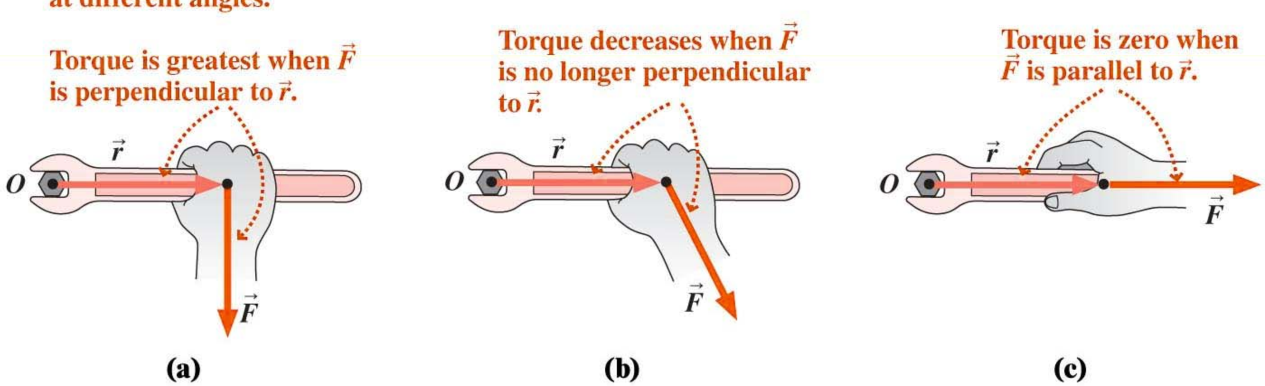
Torque

- Torque τ is the rotational analog of force, and results from the application of one or more forces.
  - Torque is relative to a chosen rotation axis.
  - Torque depends on
    - The distance from the rotation axis to the force application point.
    - The magnitude of the force $\vec{F}$.
    - The orientation of the force relative to the displacement $\vec{r}$ from axis to force application point:

$$\tau = rF\sin\theta$$



The same force is applied at different angles.



question

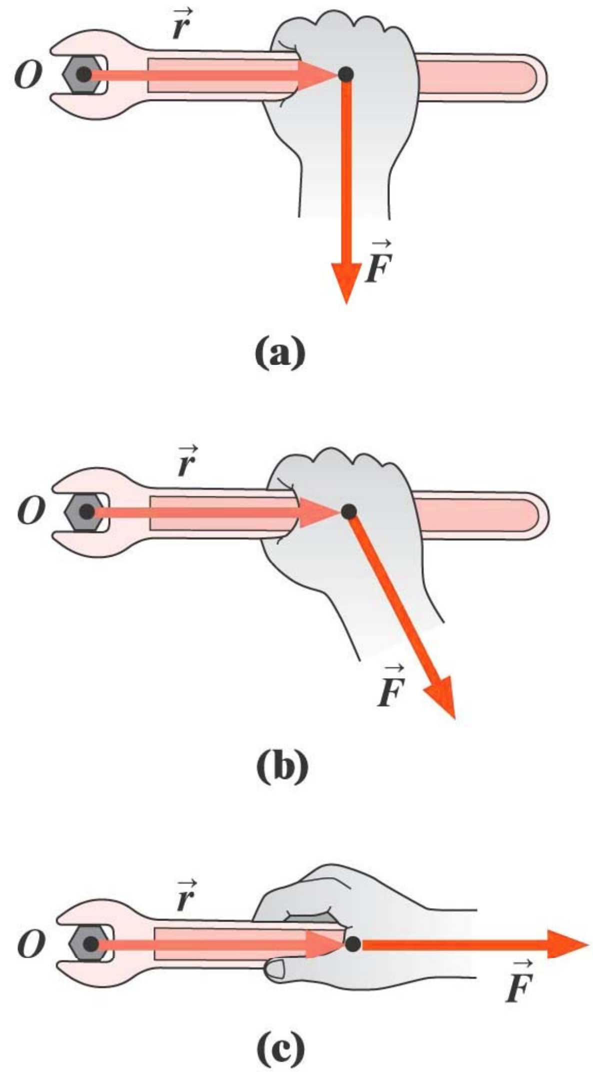
The forces in the figures all have the same magnitude. Which force produces zero torque?

A. The force in figure (a)

B. The force in figure (b)

C. The force in figure (c)

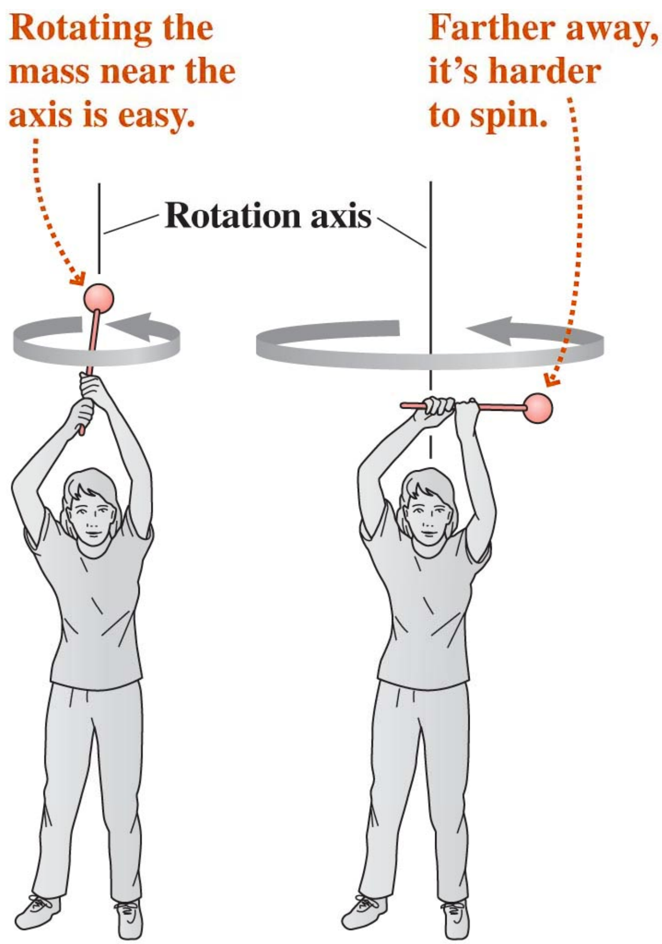
D. All of the forces produce torque



Rotational inertia and the analog of Newton's law

- Rotational inertia I is the rotational analog of mass.
  - Rotational inertia depends on mass and its distance from the rotation axis.
- Rotational acceleration, torque, and rotational inertia combine to give the rotational analog of Newton's second law:

$$\tau = I\,\alpha$$

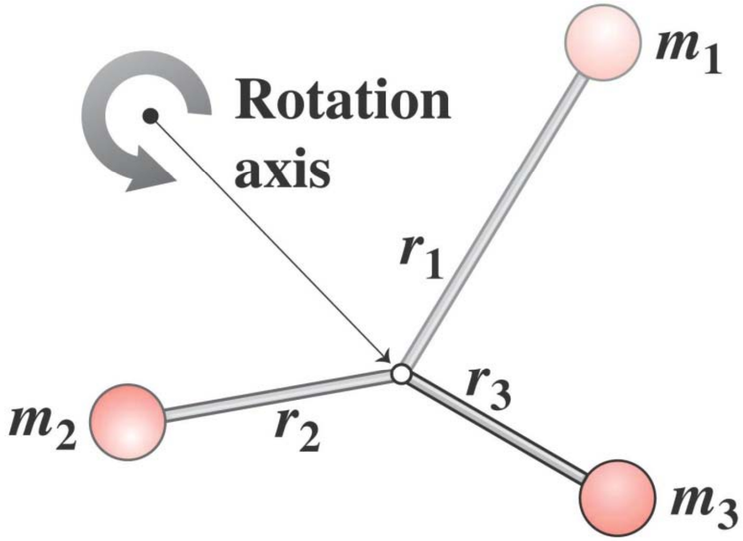


Finding rotational inertia

- For a single point mass m, rotational inertia is the product of mass with the square of the distance R from the rotation axis: $I = mR^2$.

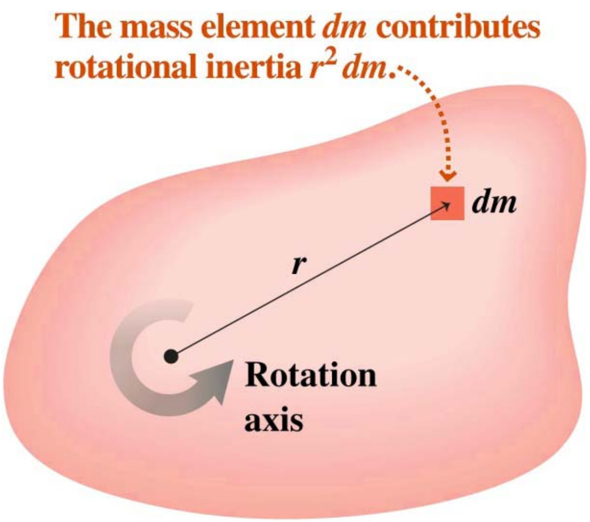
- For a system of discrete masses, the rotational inertia is the sum of the rotational inertias of the individual masses:

$$I = \sum m_i r_i^2$$



- For continuous matter, the rotational inertia is given by an integral over the distribution of matter:

$$I = \int r^2 \, dm$$



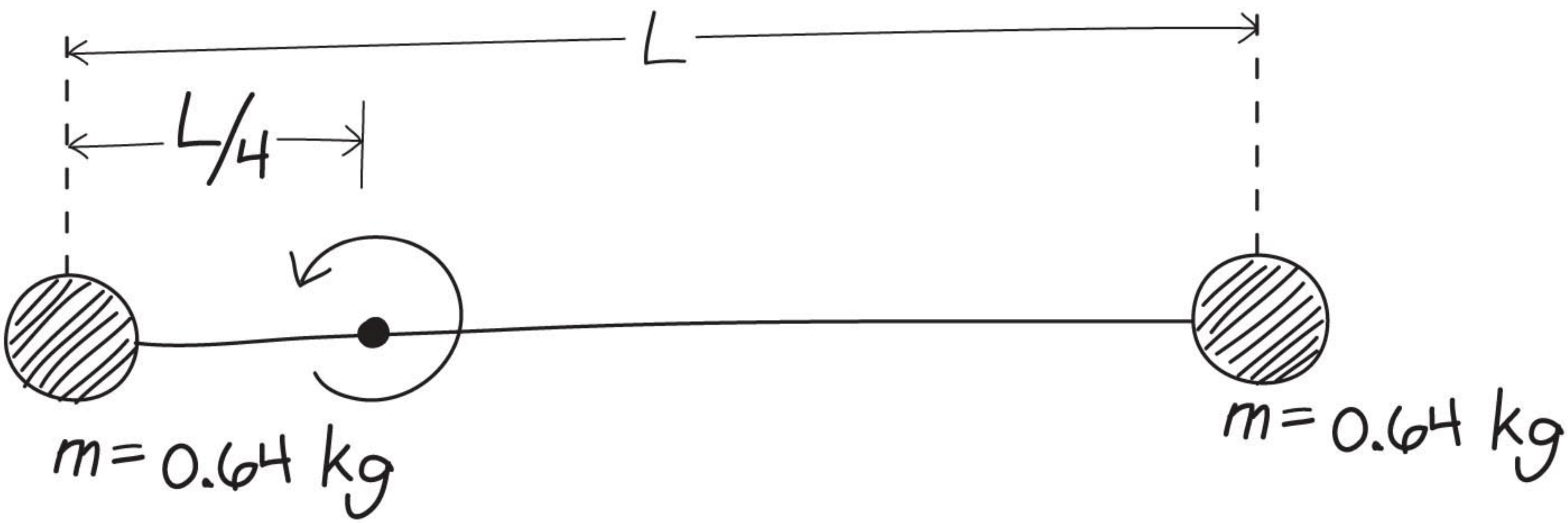
question

Consider the dumbbell in the figure. How would its rotational inertia change if the rotation axis were at the center of the rod?

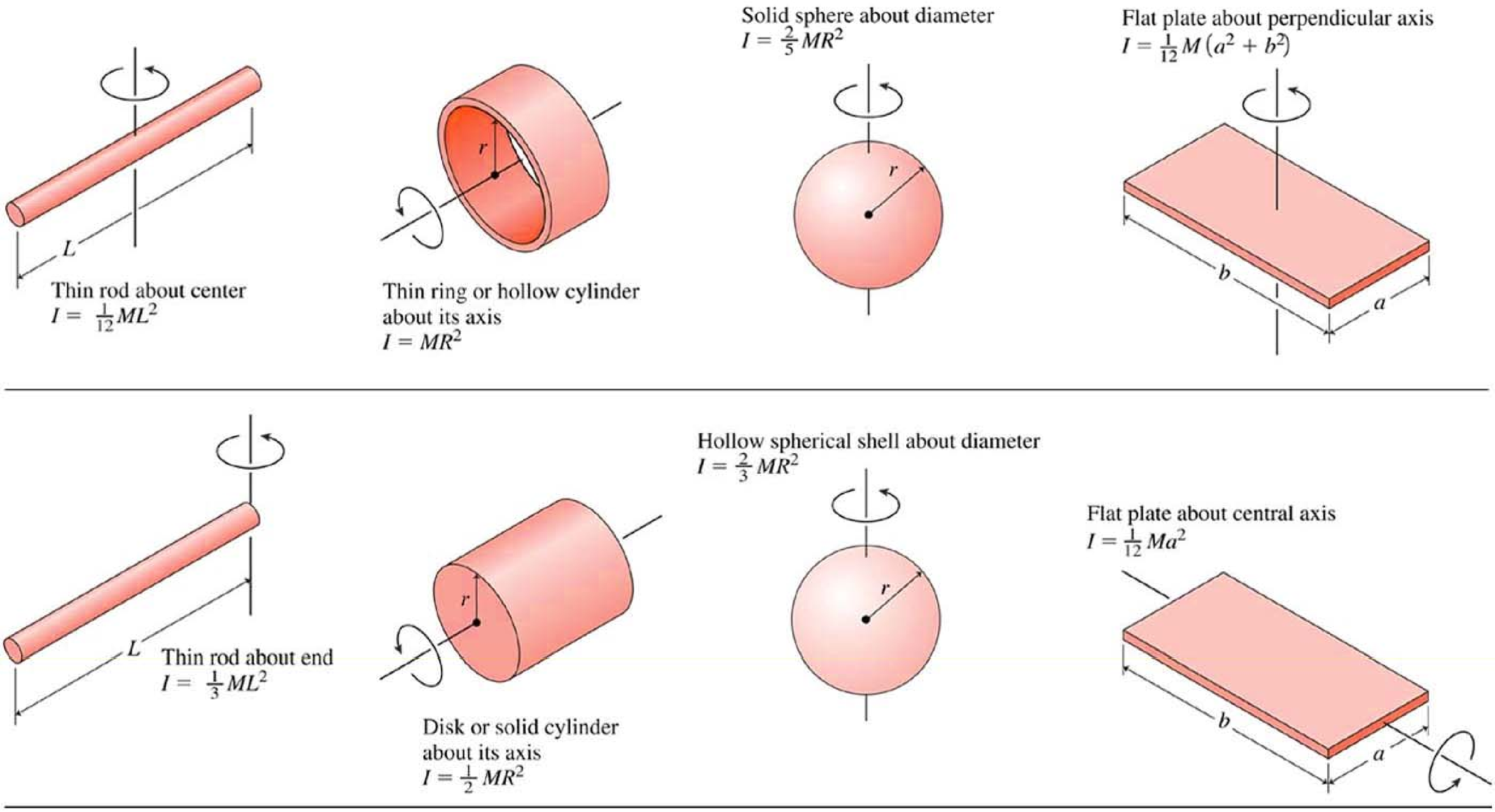
A. *I* would increase

B. *I* would decrease

C. *I* would remain the same



Rotational inertias of simple objects



question

The figure shows two identical masses *m* connected by a string that passes over a frictionless pulley whose mass is *not* negligible. One mass rests on a frictionless table while the other hangs vertically, as shown. Compare the force of tension in the horizontal and vertical sections of the string.

A. The tension in the horizontal section is greater.

B. The tension in the vertical section is greater.

C. The tensions in the two sections are equal.