Lecture 14: Conservation of Energy
Recap:

- **Work** involves force applied over distance:
  - Constant force, one dimension: \( W = F \Delta x \)
  - Varying force, one dimension: \( W = \int F(x) \, dx \)
  - In general: \( W = \int_{A}^{B} F \cdot dr \)

- The **work-energy theorem** relates the net work done on an object to the change in its **kinetic energy**, \( K = \frac{1}{2} mv^2 \):
  \[
  \Delta K = W
  \]

- **Power** is the rate at which work is done or energy is produced or used: \( P = \frac{dW}{dt} \)
In this lecture you’ll learn

• The difference between conservative and nonconservative forces

• The concept of potential energy
  • How to calculate potential energy

• Conservation of mechanical energy
  • A shortcut for solving mechanics problems
Conservative and nonconservative forces

- Examples of conservative forces include
  - Gravity
  - The static electric force
  - The force of an ideal spring
- Nonconservative forces include
  - Friction
Suppose it takes the same amount of work to push a trunk across a rough floor as it does to lift a weight the same distance straight upward. How do the amounts of work compare if the trunk and the weight are moved instead on curved paths between the same starting and ending points?

A. The two amounts of work will remain equal to each other.
B. The amount of work to move the trunk will be greater.
C. The amount of work to move the weight will be greater.
Potential energy

- The “stored work” associated with a conservative force is called **potential energy**.
  - Potential energy is stored energy that can be released as kinetic energy.
- The change in potential energy is defined as the negative of the work done by a conservative force acting over any path between two points:
  \[ \Delta U_{AB} = -\int_{A}^{B} F \cdot dr \]
  - Potential energy change is independent of path.
  - Only *changes* in potential energy matter.
  - We’re free to set the zero of potential energy at any convenient point.
Two common forms of potential energy

- **Gravitational potential energy** stores the work done against gravity:
  \[ \Delta U = mg \Delta y \]
  - Gravitational potential energy increases linearly with height \( y \).
  - This reflects the *constant* gravitational force near Earth’s surface.

- **Elastic potential energy** stores the work done in stretching or compressing springs or spring-like systems:
  \[ U = \frac{1}{2} kx^2 \]
  - Elastic potential energy increases *quadratically* with stretch or compression \( x \).
  - This reflects the *linearly increasing* spring force.
  - Here the zero of potential energy is taken in the spring’s equilibrium configuration.
Conservation of mechanical energy

- By the work-energy theorem, the change in an object’s kinetic energy equals the net work done on the object:  \( \Delta K = W_{\text{net}} \)
- When only conservative forces act, the net work is the negative of the potential-energy change:  \( W_{\text{net}} = -\Delta U \)
- Therefore when only conservative forces act, any change in potential energy is compensated by an opposite change in kinetic energy:
  \[
  \Delta K + \Delta U = 0
  \]
- Equivalently,
  \[
  K + U = \text{constant} = K_0 + U_0
  \]
- Both these equations are statements of the law of conservation of mechanical energy.
Problem-solving with conservation of energy

- **Interpret** the problem to make sure all forces are conservative, so conservation of mechanical energy applies. Identify the quantity you’re being asked to find, which may be an energy or some related quantity.

- **Develop** your solution plan by drawing the object in a situation where you can determine both its kinetic and potential energy, then again in the situation where one quantity is unknown. Also draw bar graphs showing relative sizes of the various energies.
  - Set up the equation $K + U = K_0 + U_0$

- **Evaluate** to solve for the unknown quantity, which might be an energy, a spring stretch, a velocity, etc.

- **Assess** your solution to see that your answer makes sense, has the right physical units, and is consistent with your bar graphs.
Examples

- A spring-loaded dart gun
  - What’s the dart’s speed?

  ![Initial state](image1)
  ![Final state](image2)

  *Initially there’s no kinetic energy; $K_0 = 0$.*
  *Now all the energy is kinetic; $K = \frac{1}{2}mv^2$.*

  ![Energy](image3)

  *Initially all energy is in the spring; $U_0 = \frac{1}{2}kx^2$.***
  *There’s no energy in the spring; $U = 0$.***

- A spring and gravity
  - How high does the block go?

  ![Final state](image4)

  *Initially: $U = mgh$*
  *Final state: $K = 0$*

  *$U_0 = \frac{1}{2}kx_0^2$*
  *$K_0 = 0$*

  ![Energy](image5)

  *Initially $K = 0$*
  *Final state: $U = mg$*

  ![Energy](image6)

  *$K + U = K_0 + U_0$ becomes*

  \[
  \frac{1}{2}mv^2 + 0 = 0 + \frac{1}{2}kx^2
  \]

  *So $v = \sqrt{\frac{k}{m}x}$*
  *where $x$ is the initial spring compression.*

- $K + U = K_0 + U_0$ becomes

  \[
  0 + mgh = 0 + \frac{1}{2}kx^2
  \]

  *So $h = \frac{kx^2}{2mg}$*