Physics 1501

Fall 2008

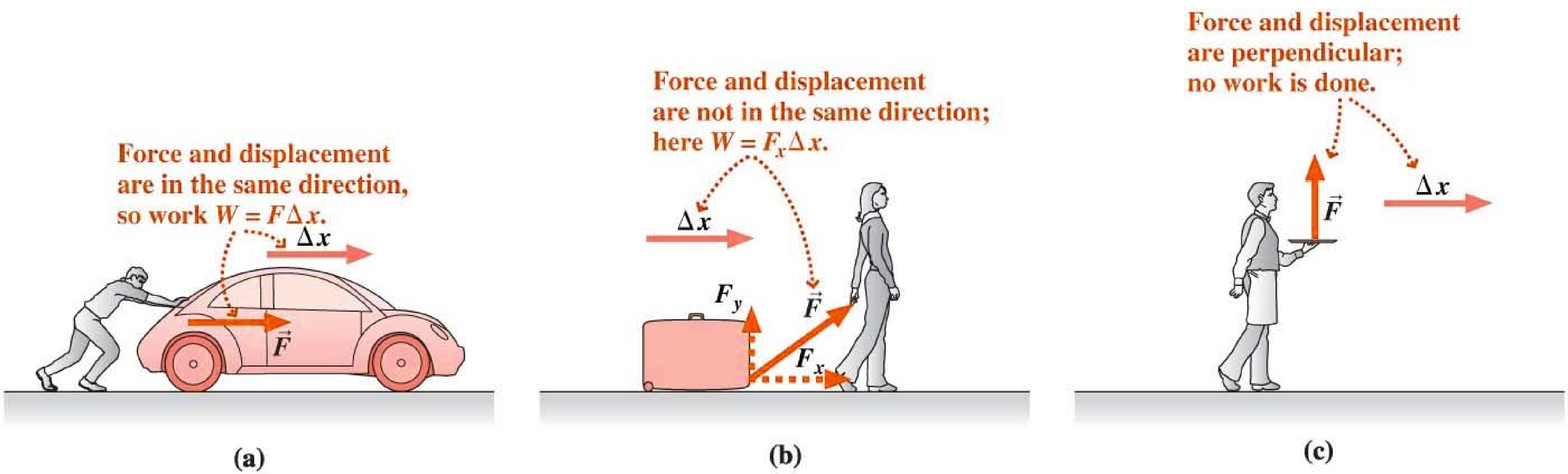
Mechanics, Thermodynamics, Waves, Fluids

Lecture 13: Work, Energy, Power II

Recap: Work - a measure of force applied over distance

In one dimension: $W = F_x \Delta x$

More generally, work depends on the *component of force in the direction of motion*:



Recap: the scalar product

Work is conveniently characterized using the *scalar product*, a way of combining two vectors to produce a scalar that depends on the vectors' magnitudes and the angle between them.

The scalar product of any two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

where A and B are the magnitudes of the vectors and θ is the angle between them.

With vectors in component form, $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ **and** $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, **the scalar product can be written**

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

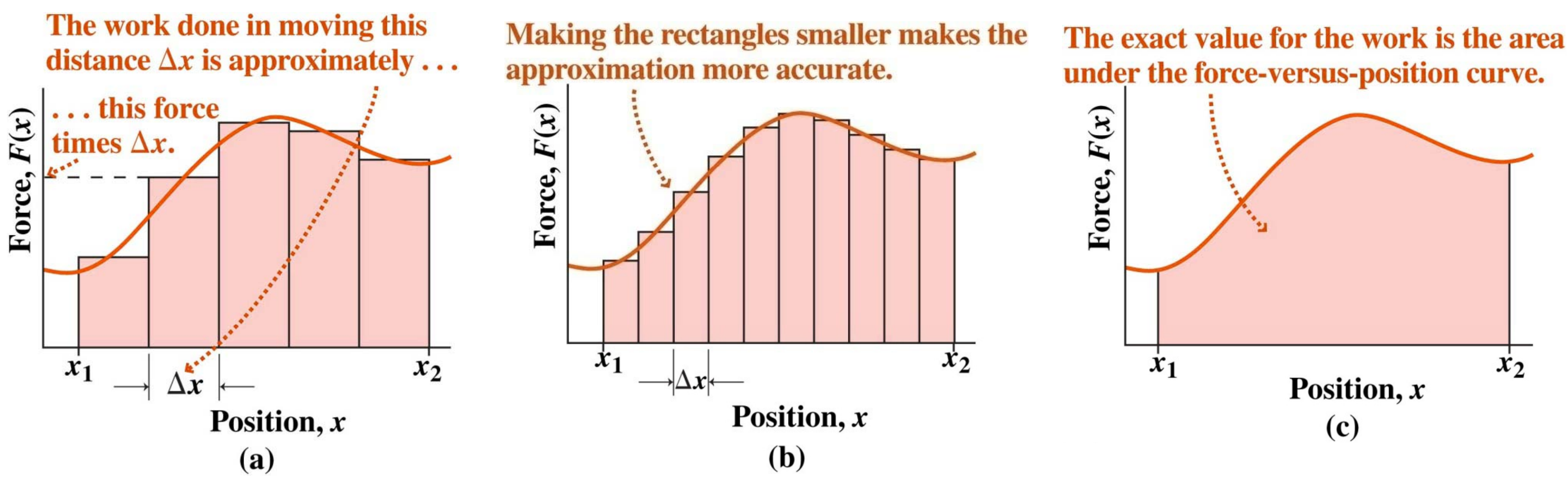
- **Work is the scalar product of force with displacement:**

$$W = \vec{F} \cdot \Delta\vec{r}$$

Recap: work done by a varying force

When a force varies with position, it's necessary to integrate to calculate the work done.

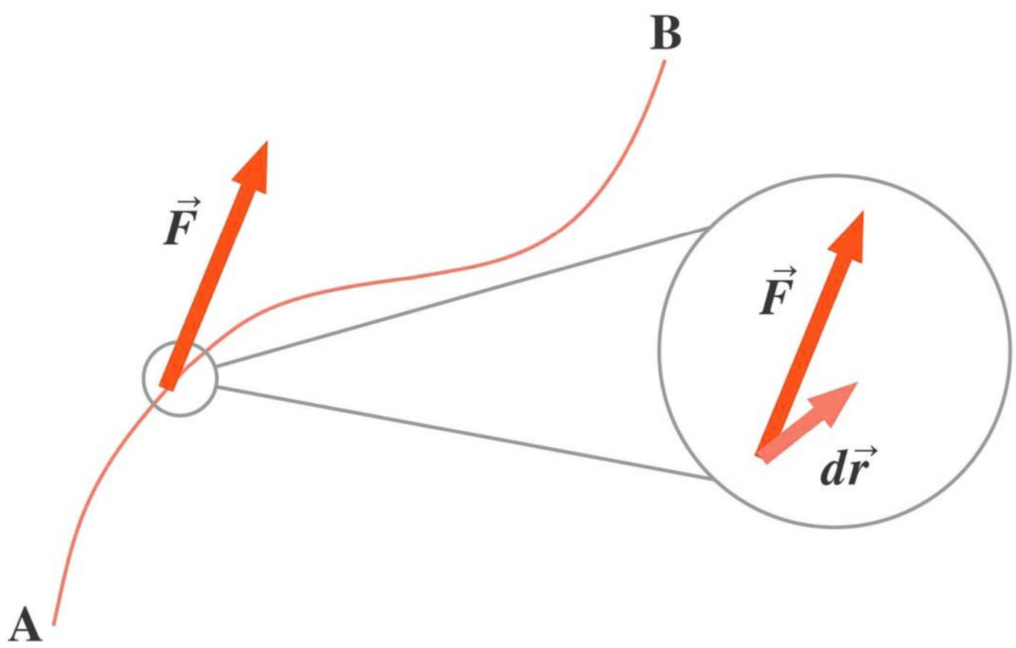
Geometrically, the work is the area under the force-versus-position curve.



Recap: varying force in multiple dimensions

In the most general case, an object moves on an arbitrary path subject to a force whose magnitude and whose direction relative to the path may vary with position.

In that case the integral for the work becomes a **line integral**, the limit of the sum of scalar products of infinitesimally small displacements with the force at each point.



$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

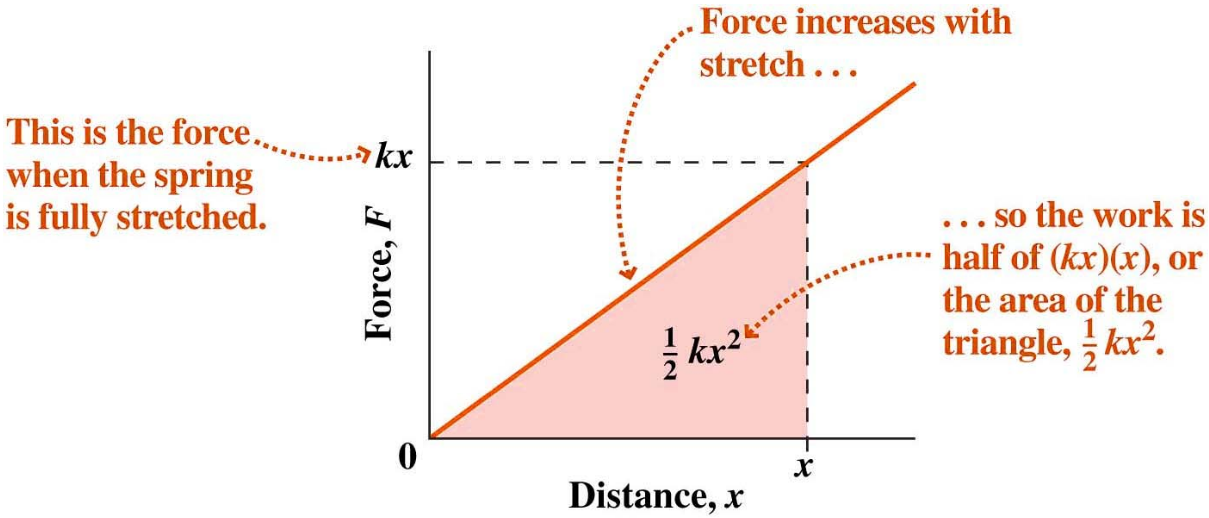
Recap: work done in stretching a spring

A spring exerts a force $F_{\text{spring}} = -kx$.

Therefore the agent stretching a spring exerts a force $F = +kx$, and the work the agent does is

$$W = \int_0^x F(x)\,dx = \int_0^x kx\,dx = \tfrac{1}{2}kx^2\Big|_0^x = \tfrac{1}{2}kx^2 - \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}kx^2$$

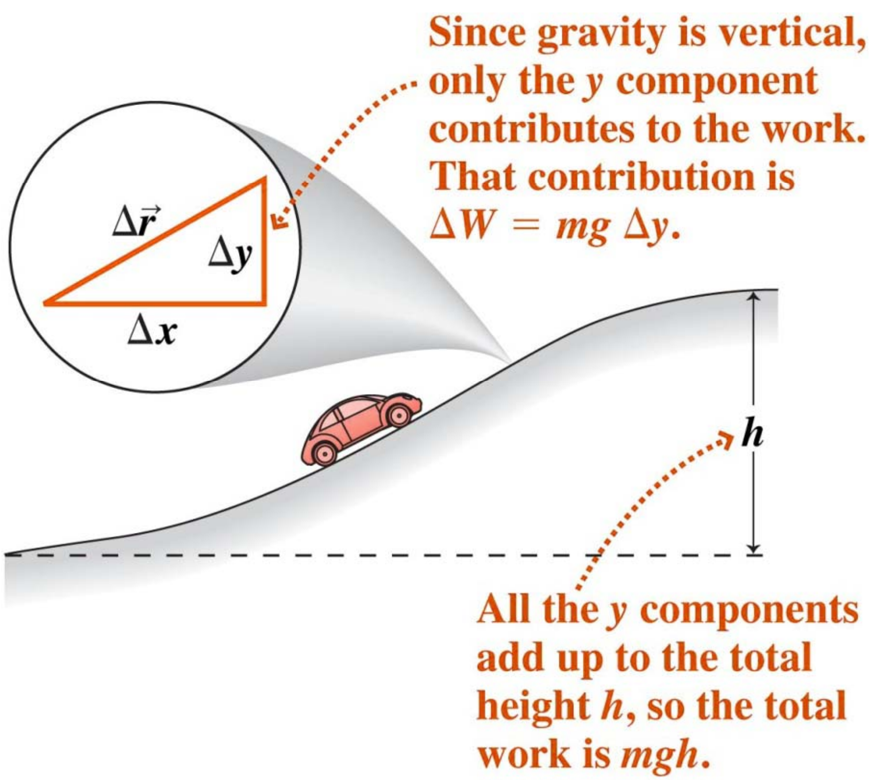
- **In this case the work is the area under the triangular force-versus-position curve:**



Recap: work done against gravity

The work done by an agent lifting an object of mass m against gravity depends only on the vertical distance h:

$$W = mgh$$



- **The work is positive if the object is raised and negative if it's lowered.**

Work and net work

The work *you* do in moving an object involves only the force *you* apply:

But there may be other forces acting on the object as well.

The **net work** is the work done by all the forces acting—that is, the work done by the net force.

Example:

Lift an object at constant speed, and you do work mgh.

But gravity, acting downward, does work $-mgh$.

So the net work in this case is zero.

The work-energy theorem

Applying Newton's second law to the net work done on an object results in the **work-energy theorem:**

$$W_{\text{net}} = \int F_{\text{net}}\,dx = \int ma\,dx = \int m\frac{dv}{dt}\,dx = \int m\frac{dx}{dt}\,dv = \int mv\,dv$$

- Evaluating the last integral between initial and final velocities v_1 and v_2 gives

$$W_{\text{net}} = \int_{v_1}^{v_2} mv\,dv = \tfrac{1}{2}mv^2\Big|_{v_1}^{v_2} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

- So the quantity $\tfrac{1}{2}mv^2$ changes only when net work is done on an object, and the change in this quantity is equal to the net work.

Kinetic energy and the work-energy theorem

- The quantity $\frac{1}{2}mv^2$ is called kinetic energy, K. Kinetic energy is a kind of energy associated with motion:

The kinetic energy K of an object of mass m moving at speed v is

$$K = \frac{1}{2}mv^2$$

- Then the work-energy theorem states that the change in an object's kinetic energy is equal to the net work done on the object:

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = W_{net}$$

Power and energy

- Power is the *rate* at which work is done or at which energy is used or produced. If work ΔW is done in time Δt, then the average power over this time is

$$\overline{P} = \frac{\Delta W}{\Delta t} \quad \text{(average power)}$$

- When the rate changes continuously, the instantaneous power is

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

- Power is measured in watts (W), with 1 W = 1 J/s.

- Total work or energy follows from power by multiplying (for constant power) or integrating (for varying power):

$$W = P\,\Delta t \quad \text{or} \quad W = \int_{t_1}^{t_2} P\,dt$$

Summary

- **Work** involves force applied over distance:
  - Constant force, one dimension: $W = F_x \Delta x$
  - Varying force, one dimension: $W = \int F(x)dx$
  - In general: $W = \int_{\mathrm{A}}^{\mathrm{B}} \vec{F} \cdot d\vec{r}$
- The **work-energy theorem** relates the net work done on an object to the change in its **kinetic energy**, $K = \frac{1}{2}mv^2$:

$$\Delta K = W$$

- **Power** is the rate at which work is done or energy is produced or used: $P = dW/dt$