Lecture 11: using Newton’s laws III
Circular Motion

• Problems involving circular motion are no different from other Newton’s law problems.

• Identify the forces, draw a freebody diagram, write Newton’s law.

• Here the acceleration has magnitude $v^2/r$ and points toward the center of the circle.

A ball whirling on a string:

Freebody diagram:

Newton’s law:
$$T + F_g = ma$$

In components:
$$x: \quad T \cos \theta = \frac{mv^2}{L \cos \theta}$$
$$y: \quad T \sin \theta - mg = 0$$

Solve for the ball’s speed:
$$v = \sqrt{\frac{TL \cos^2 \theta}{m}} = \sqrt{\frac{(mg / \sin \theta)L \cos^2 \theta}{m}} = \sqrt{\frac{gL \cos^2 \theta}{\sin \theta}}$$
Loop-the-Loop!

- The two forces acting on the car are gravity and the normal force.
- Gravity is always downward, and the normal force is perpendicular to the track.
- Here the two are at right angles:
  - The normal force acts perpendicular to the car’s path, keeping its direction of motion changing.
  - Gravity acts opposite the car’s velocity, slowing the car.
Loop-the-Loop!

- Now both forces are downward:
  - For the car to stay in contact with the track, the normal force must be greater than zero.
  - So the minimum speed is the speed that lets the normal force get arbitrarily close to zero at the top of the loop.
  - Then gravity alone provides the force that keeps the car in circular motion.
Loop-the-Loop!

- Therefore Newton’s law has a single component, with the gravitational force $mg$ providing the acceleration $v^2/r$ that holds the car in its circular path:

$$F = ma \quad \rightarrow \quad mg = \frac{mv^2}{r}$$

- Solving for the minimum speed at the loop top gives $v = \sqrt{gr}$.
- Slower than this at the top, and the car will leave the track!
- Since this result is independent of mass, car and passengers will all remain on the track as long as $v \geq \sqrt{gr}$. 
Friction

- **Friction** is a force that opposes the relative motion of two contacting surfaces.

- **Static friction** occurs when the surfaces aren’t in motion; its magnitude is $f_s \geq \mu_s n$, where $n$ is the normal force between the surfaces and $\mu_s$ is the coefficient of static friction.

- **Kinetic friction** occurs between surfaces in motion; its magnitude is $f_k = \mu_k n$.

Friction is important in walking, driving and a host of other applications:
Solving Problems with Friction

- Problems with friction are like all other Newton’s law problems.
- Identify the forces, draw a freebody diagram, write Newton’s law.
- You’ll need to relate the force components in two perpendicular directions, corresponding to the normal force and the frictional force.

A braking car: What’s the acceleration?

Newton’s law: \( \vec{F}_g + \vec{n} + \vec{f}_f = ma \)

In components:
- \( x: \quad -\mu n = ma_x \)
- \( y: \quad -mg + n = 0 \)

Solve for \( a \):
\( y \) equation gives \( n = mg \),
so \( x \) equation gives \( a_x = -\frac{\mu n}{m} = -\mu g \)
Summary

• All Newton’s law problems are the same.
• They’re handled by
  • Identifying all the forces acting on the object or objects of interest.
  • Drawing a freebody diagram.
  • Writing Newton’s law in vector form:
    • Equating the net force to the mass times the acceleration.
  • Establishing a coordinate system.
  • Writing Newton’s law in components.
• Solving for the quantities of interest.