Physics 1501

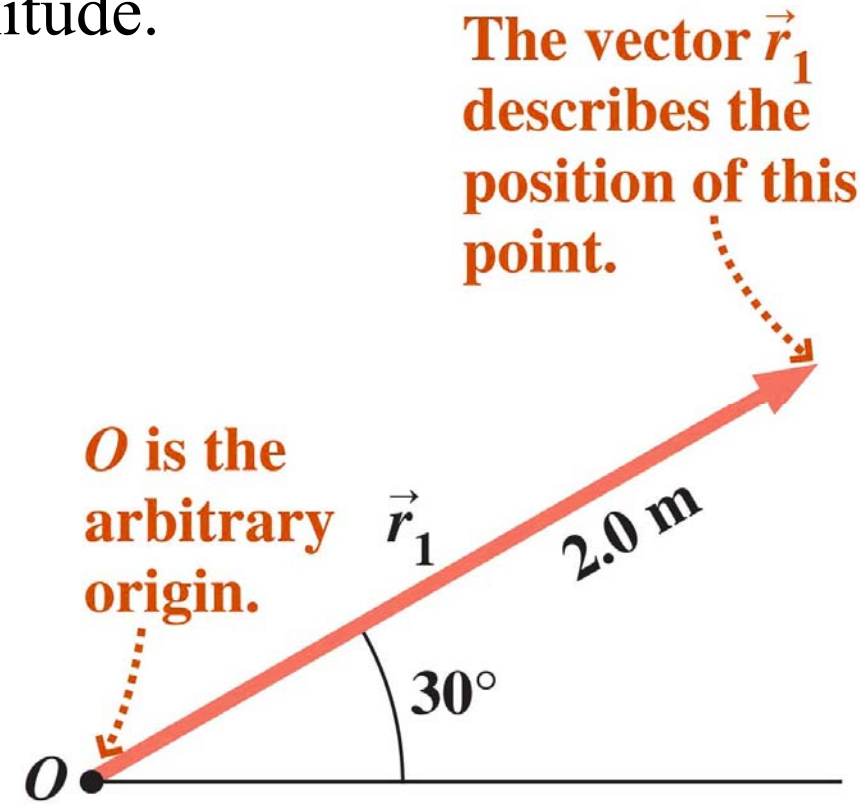
Fall 2008

Mechanics, Thermodynamics, Waves, Fluids

Lecture 5: motion in two and three dimensions II

Recap: vectors

- A **vector** is a quantity that has both magnitude and direction.
  - In two dimensions it takes two numbers to specify a vector.
  - In three dimensions it takes three numbers.
  - A vector can be represented by an arrow whose length corresponds to the vector's magnitude.
- Position is a vector quantity.
  - An object's position is specified by giving its distance from an origin and its direction relative to an axis.
  - Here $\vec{r}_1$ describes a point 2.0 m from the origin at a 30° angle to the axis.

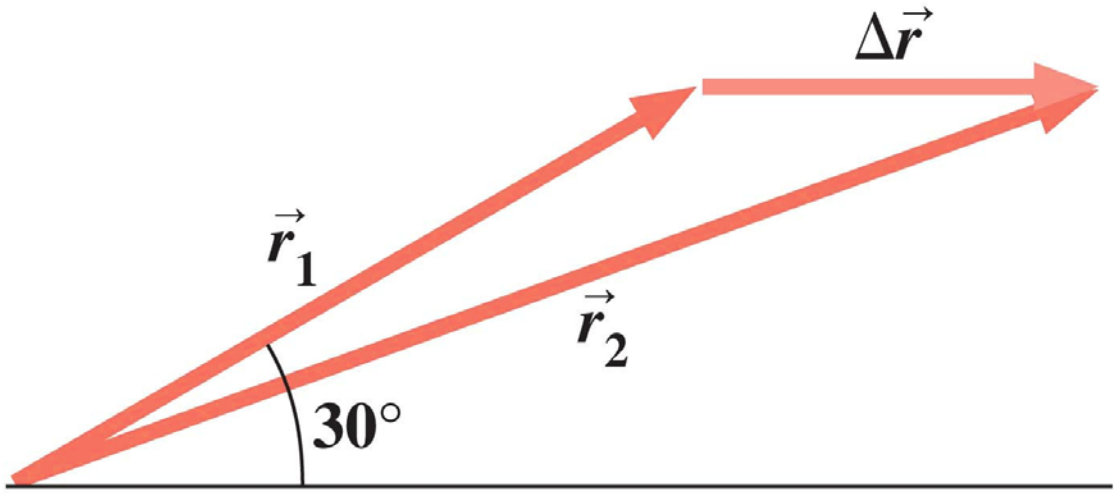


Recap: adding vectors

- To add vectors graphically, place the tail of the first vector at the head of the second.
  - Their sum is then the vector from the tail of the first vector to the head of the second.

- Here $\vec{r}_2$ is the sum of $\vec{r}_1$ and $\Delta\vec{r}$.

$$\vec{r}_2 = \vec{r}_1 + \Delta\vec{r}$$

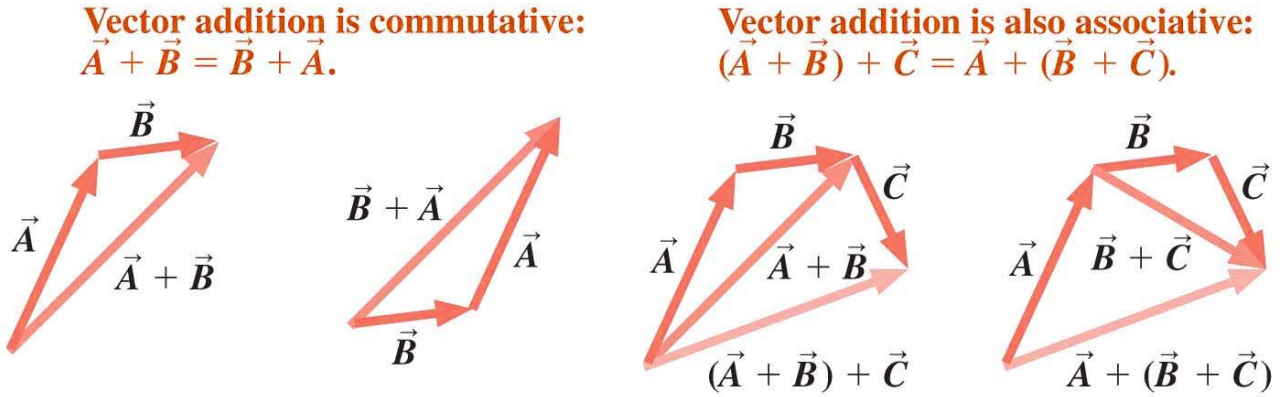


Recap: vector operations

- To multiply a vector by a scalar, multiply the vector's magnitude by the scalar.
  - For a positive scalar the direction is unchanged.
  - For a negative scalar the direction reverses.
- To subtract vectors, add the negative of the second vector to the first:

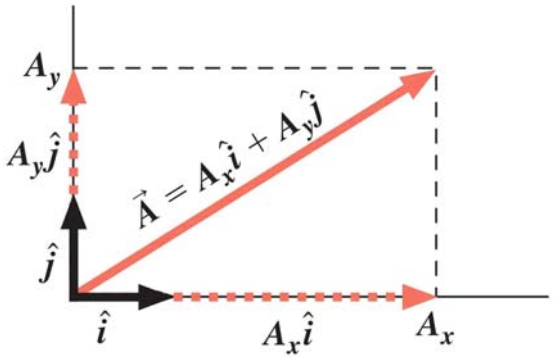
$$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

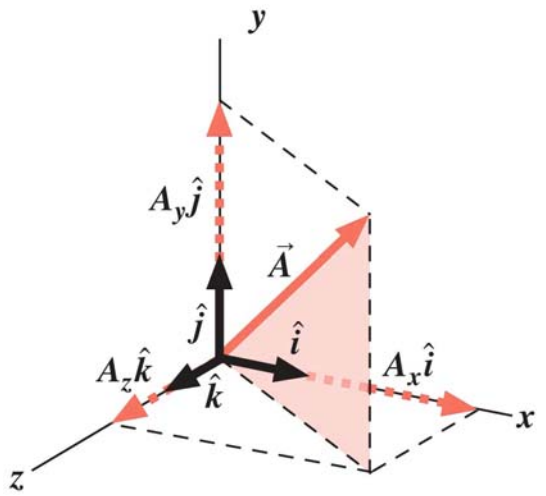
- Vector arithmetic is commutative and associative:



Recap: unit vectors

- Unit vectors have magnitude 1, no units, and point along the coordinate axes.
  - They're used to specify direction in compact mathematical representations of vectors.
  - Unit vectors in the *x*, *y*, and *z* directions are designated $\hat{i}$, $\hat{j}$, and $\hat{k}$.
- Any vector in two dimensions can be written as a linear combination of $\hat{i}$ and $\hat{j}$.
- Any vector in three dimensions can be written as a linear combination of $\hat{i}$, $\hat{j}$, and $\hat{k}$.





Recap: velocity and acceleration vectors

- Velocity is the rate of change of position.
  - The average velocity over a time interval Δt is the change in the position vector divided by the time.
    - Here dividing by Δt means multiplying by the scalar $1/\Delta t$:

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$$

- Instantaneous velocity is the time derivative of position:

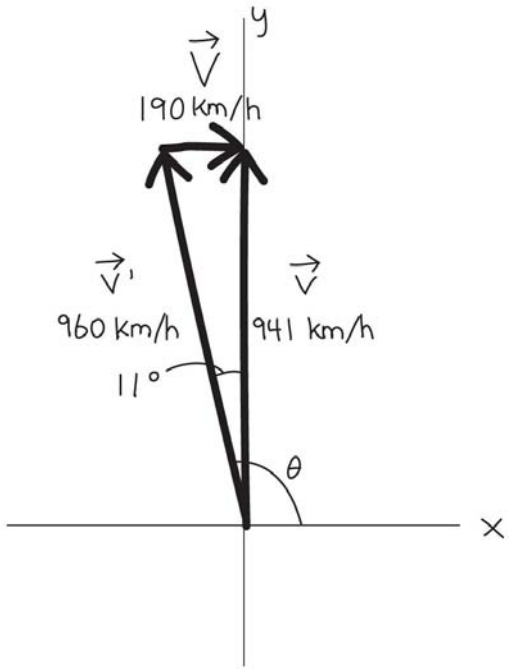
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

- Acceleration is the rate of change of velocity:

$$\text{Average: } \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \qquad \text{Instantaneous: } \vec{a} = \frac{d\vec{v}}{dt}$$

Relative motion

- An object moves with velocity $\vec{v}'$ relative to one frame of reference.
- That frame moves at $\vec{V}$ relative to a second reference frame.
- Then the velocity of the object relative to the second frame is $\vec{v} = \vec{v}' + \vec{V}$.
- Example:
  - A jetliner flies at 960 km/h relative to the air, heading northward. There's a wind blowing eastward at 190 km/h. In what direction should the plane fly?
  - The vector diagram identifies the quantities in the equation, and shows that the angle is 11°.



Constant acceleration

- With constant acceleration, the equations for one-dimensional motion apply independently in each direction.
  - The equations take a compact form in vector notation.
  - Each equation stands for two or three separate equations.

$$\vec{v} = \vec{v}_0 + \vec{a}\,t$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

question

- An object is moving initially in the +*x* direction. Which of the following accelerations, all acting for the same time interval, will cause the greatest change in its speed?

A. $10\hat{j}$ m/s^2
B. $2\hat{i} - 8\hat{j}$ m/s^2
C. $10\hat{i}$ m/s^2
D. $10\hat{i} + 5\hat{j}$ m/s^2

Projectile motion

- Motion under the influence of gravity near Earth's surface has essentially constant acceleration $\vec{g}$ whose magnitude is $g = 9.8\ \text{m/s}^2$, and whose direction is downward.
  - Such motion is called **projectile motion.**
  - Equations for projectile motion, in a coordinate system with y axis vertically upward:

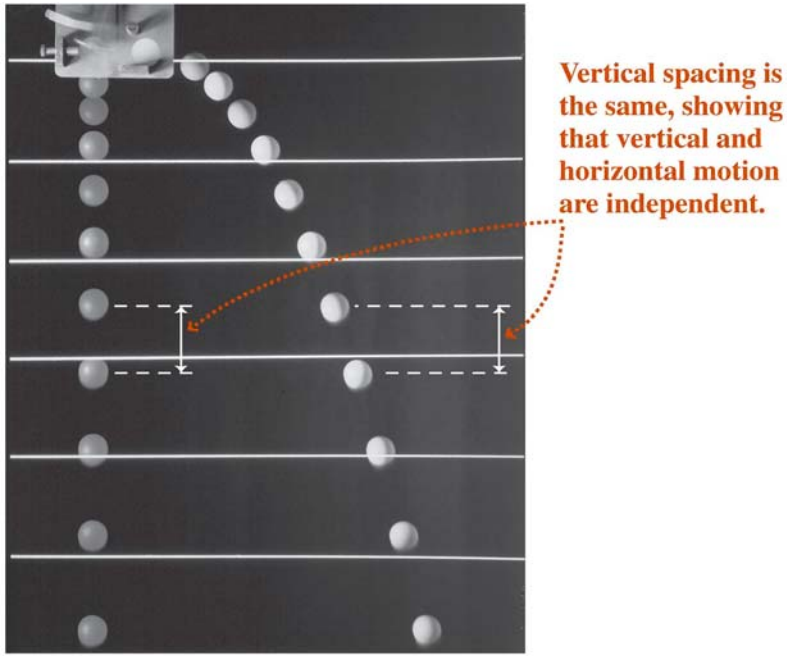
$$v_x = v_{x0}$$

$$v_y = v_{y0} - gt$$

$$x = x_0 + v_{x0}t$$

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$

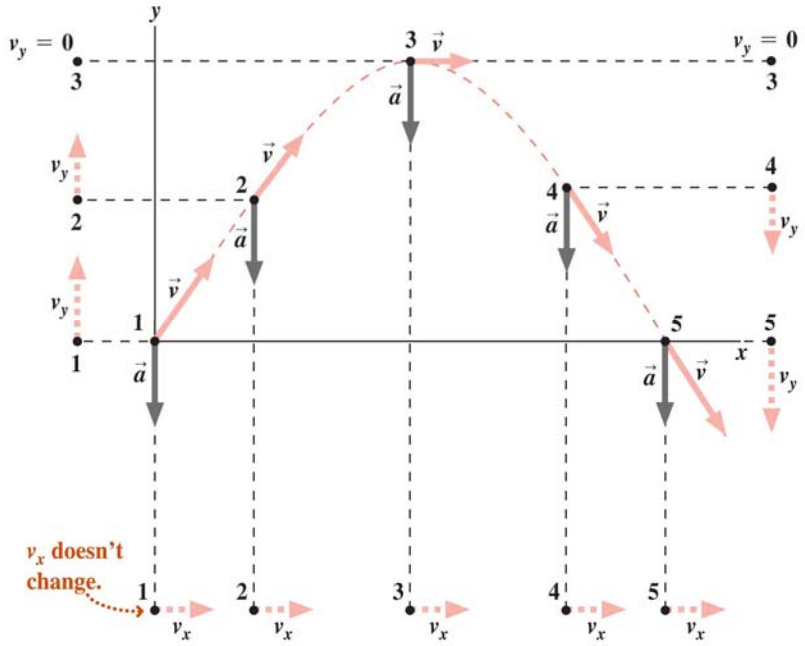
  - Horizontal and vertical motions are independent:

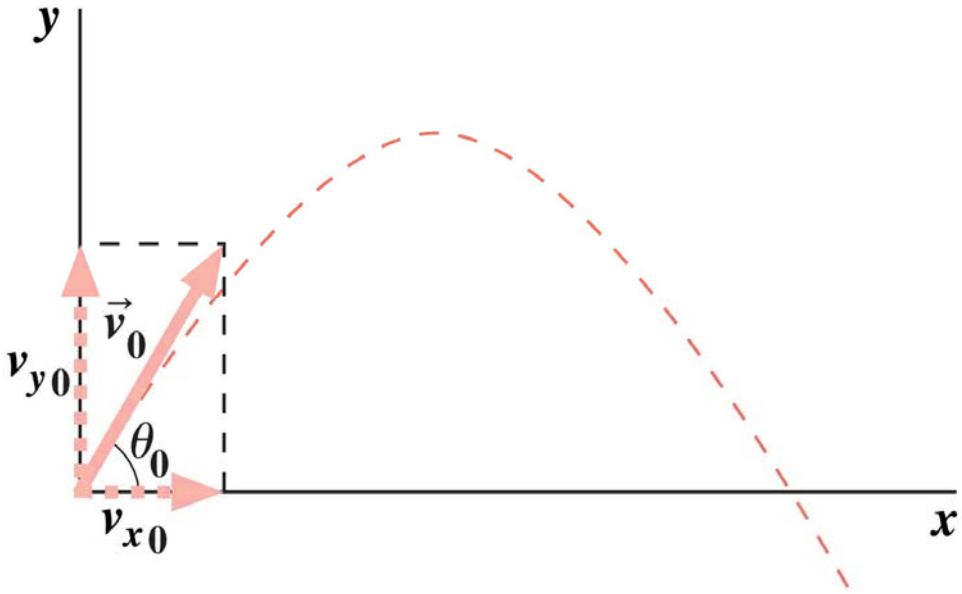


Projectile trajectories

- The trajectory of an object in projectile motion is a parabola, unless the object has no horizontal component of motion.
  - Horizontal motion is unchanged, while vertical motion undergoes downward acceleration:
  - Equation for the trajectory:

$$y = x\tan\theta_0 - \frac{g}{2v_0^2\cos^2\theta_0}x^2$$



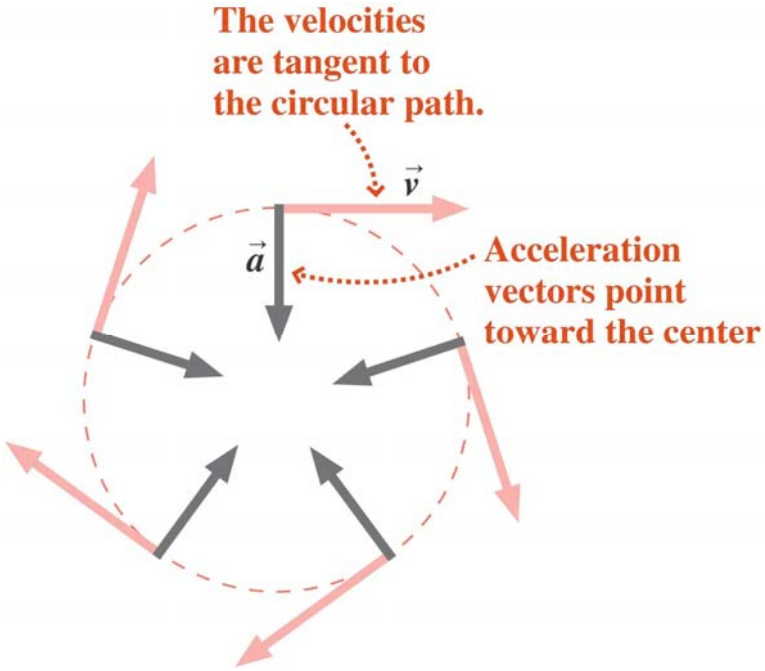


Uniform circular motion

- When an object moves in a circular path of radius r at constant speed v, its acceleration has magnitude

$$a = \frac{v^2}{r}$$

  - The acceleration vector points toward the center of the circle.
  - Since the direction of the acceleration keeps changing, this is *not* constant acceleration.



question

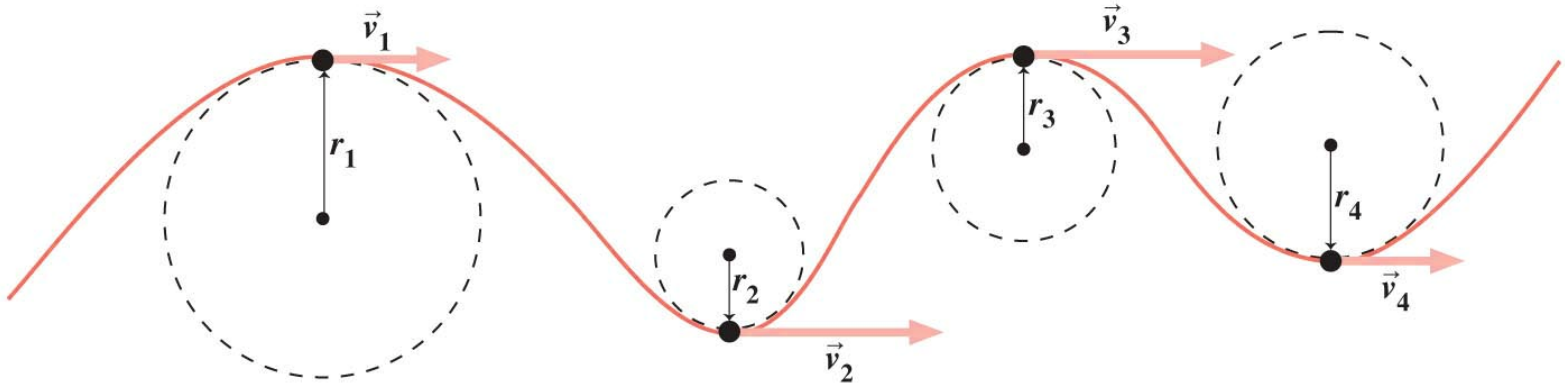
- The figure shows velocity vectors for four points on a noncircular path. Choose the correct order, from smallest to largest, of the centripetal accelerations at these points given $v_1 = v_4$ and $v_2 = v_3$.

A. $a_1 > a_4 > a_3 > a_2$

B. $a_2 > a_3 > a_4 > a_1$

C. $a_3 > a_2 > a_1 > a_4$

D. $a_2 > a_3 > a_1 > a_4$



Summary

- In two and three dimensions, position, velocity, and acceleration become vector quantities.
  - Velocity is the rate of change of position: $\vec{v} = \frac{d\vec{r}}{dt}$
  - Acceleration is the rate of change of velocity: $\vec{a} = \frac{d\vec{v}}{dt}$
- In general, acceleration changes both the magnitude and direction of the velocity.
- Projectile motion results from the acceleration of gravity.
- In uniform circular motion, the acceleration has magnitude v^2/r and points toward the center of the circular path.