Physics 1501

Fall 2008

Mechanics, Thermodynamics, Waves, Fluids

Lecture 4: motion in two and three dimensions I

Recap: Velocity

- **Velocity** is the rate of change of position.
  - **Average velocity** over a time interval Δt is defined as the displacement divided by the time:

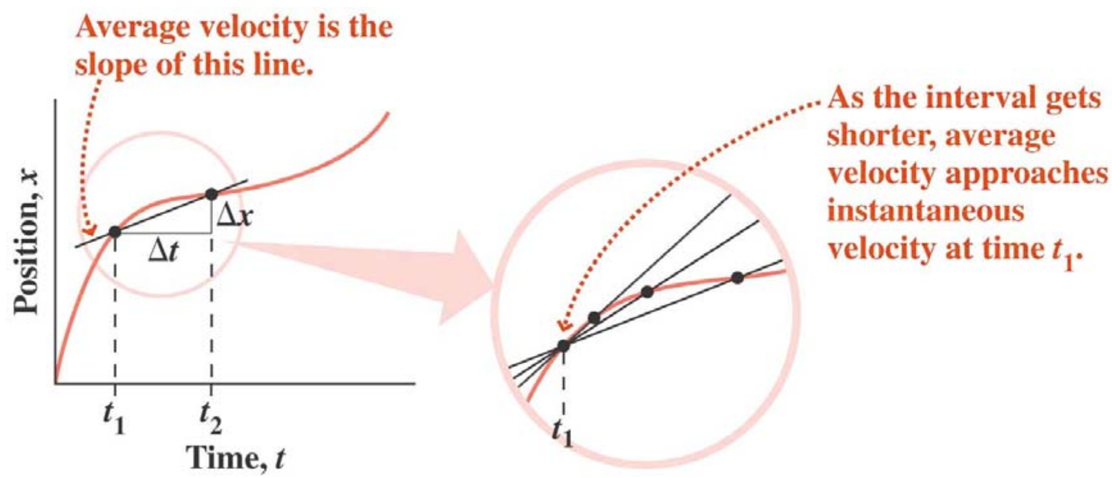
$$\bar{v} = \frac{\Delta x}{\Delta t}$$

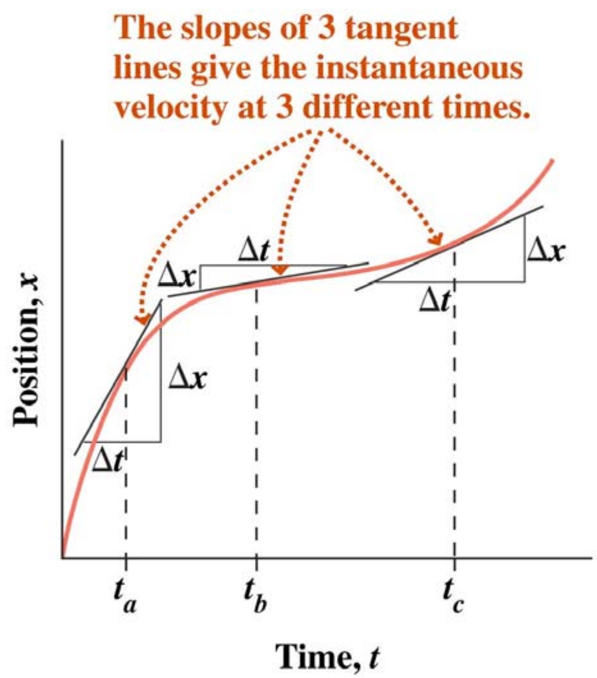
  - **Instantaneous velocity** is the limit of the average velocity as the time interval becomes arbitrarily short:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

    - In calculus, this limiting procedure defines the **derivative** *dx/dt.*
  - **Speed** is the magnitude of velocity.

- Velocity is the slope of the position-versus-time curve.





Recap: Acceleration

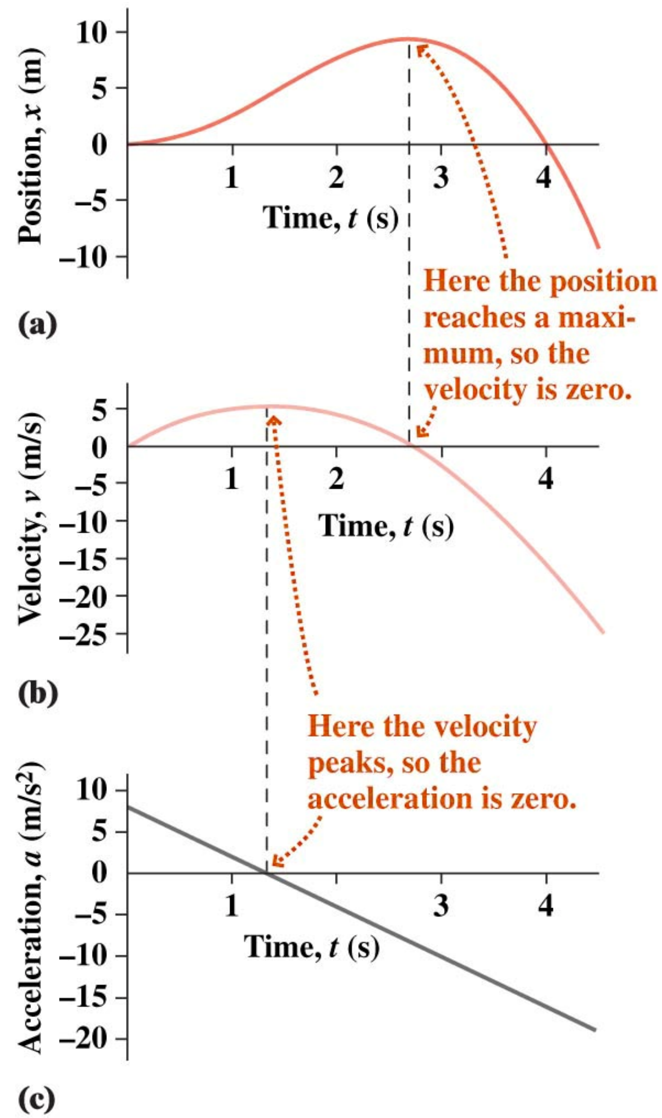
- **Acceleration** is the rate of change of velocity.
  - **Average velocity** over a time interval Δt is defined as the change in velocity divided by the time:

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

  - **Instantaneous acceleration** is the limit of the average acceleration as the time interval becomes arbitrarily short:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- Acceleration is the slope of the velocity-versus-time curve.



Recap: Constant acceleration

- When acceleration is constant, then position, velocity, acceleration, and time are related by

$$v = v_0 + at$$

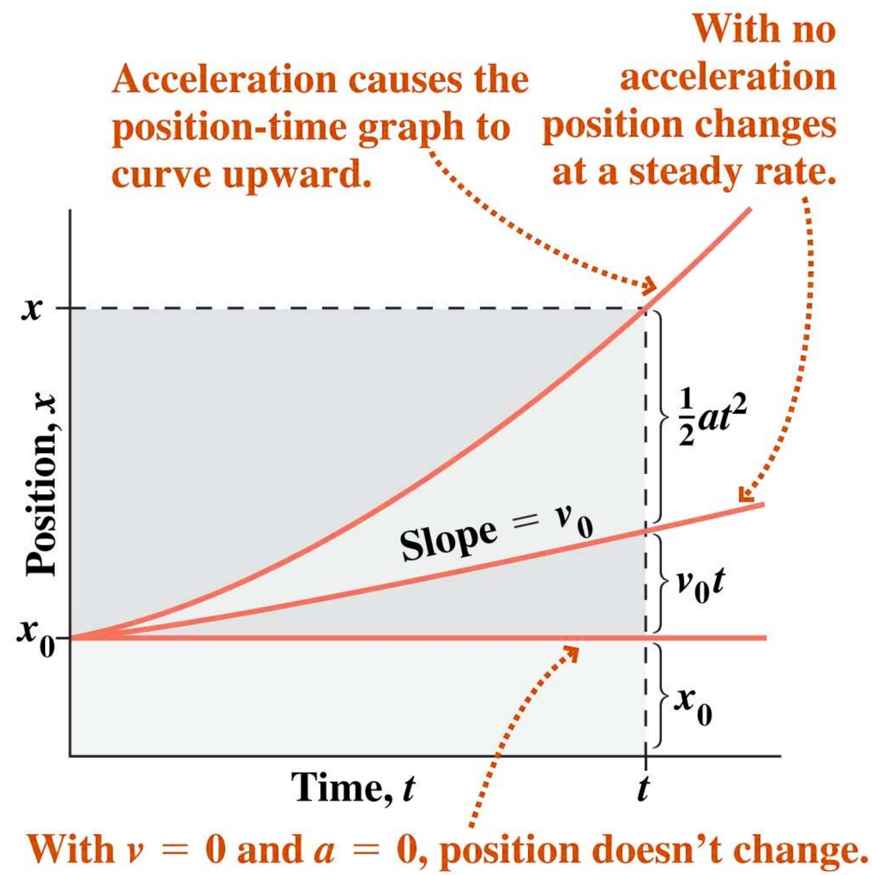
$$x = x_0 + \tfrac{1}{2}(v_0 + v)t$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

where x_0 and v_0 are initial values at time $t = 0$, and x and v are the values at an arbitrary time t.

- With constant acceleration
  - Velocity is a linear function of time
  - Position is a quadratic function of time



Recap: the acceleration of gravity

- The acceleration of gravity at any point is exactly the same for all objects, regardless of mass.
- Near Earth's surface, the value of the acceleration is essentially constant at $g = 9.8\ \text{m/s}^2$.
- Therefore the equations for constant acceleration apply:
  - In a coordinate system with y axis upward, they read

$$v = v_0 - gt$$

$$y = y_0 + \tfrac{1}{2}(v_0 + v)t$$

$$y = y_0 + v_0 t - \tfrac{1}{2}gt^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

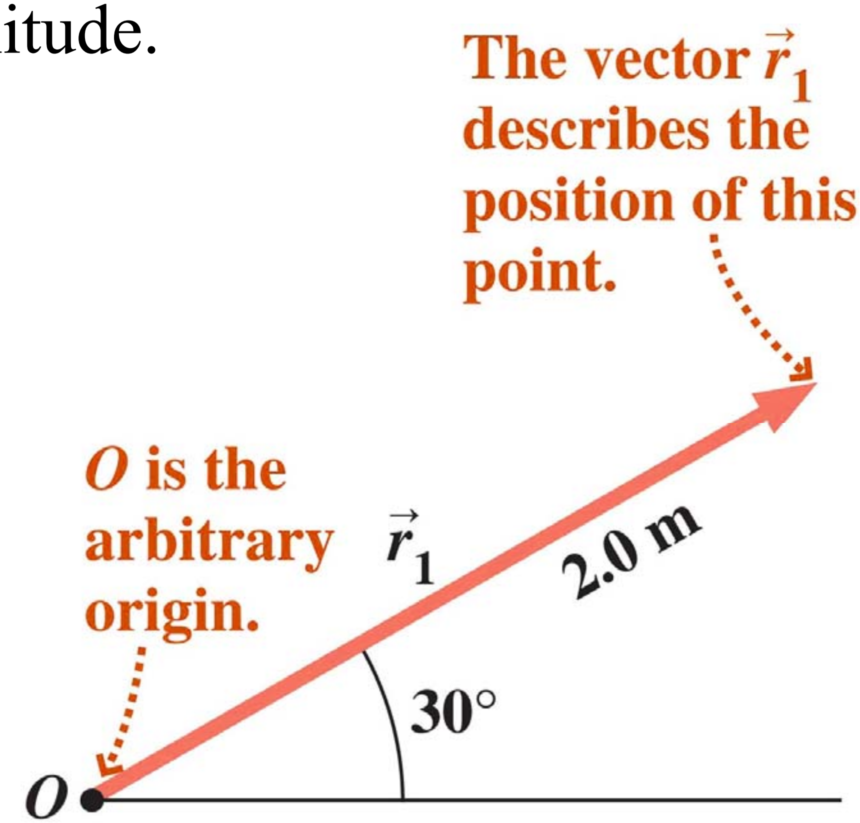
This strobe photo of a falling ball shows increasing spacing resulting from the acceleration of gravity.

In this lecture you'll learn

- To describe position, velocity, and acceleration in three-dimensional space using the language of vectors
- To manipulate vectors algebraically
- To transform velocities to different reference frames
- To solve problems involving constant acceleration in two dimensions
  - Including projectile motion due to the constant acceleration of gravity near Earth's surface
- To evaluate acceleration in circular motion

Vectors

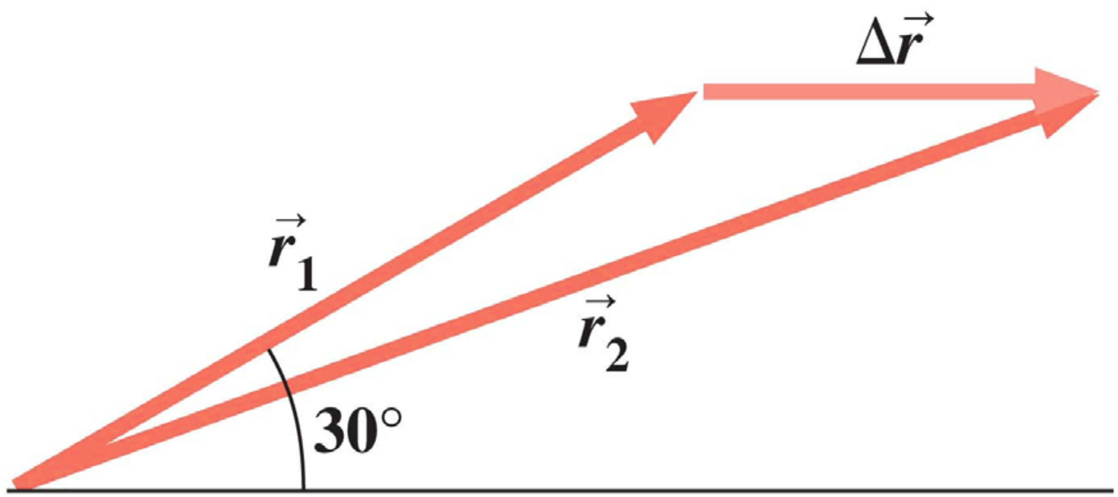
- A **vector** is a quantity that has both magnitude and direction.
  - In two dimensions it takes two numbers to specify a vector.
  - In three dimensions it takes three numbers.
  - A vector can be represented by an arrow whose length corresponds to the vector's magnitude.
- Position is a vector quantity.
  - An object's position is specified by giving its distance from an origin and its direction relative to an axis.
  - Here $\vec{r}_1$ describes a point 2.0 m from the origin at a 30° angle to the axis.



Adding vectors

- To add vectors graphically, place the tail of the first vector at the head of the second.
  - Their sum is then the vector from the tail of the first vector to the head of the second.
  - Here $\vec{r}_2$ is the sum of $\vec{r}_1$ and $\Delta\vec{r}$.

$$\vec{r}_2 = \vec{r}_1 + \Delta\vec{r}$$



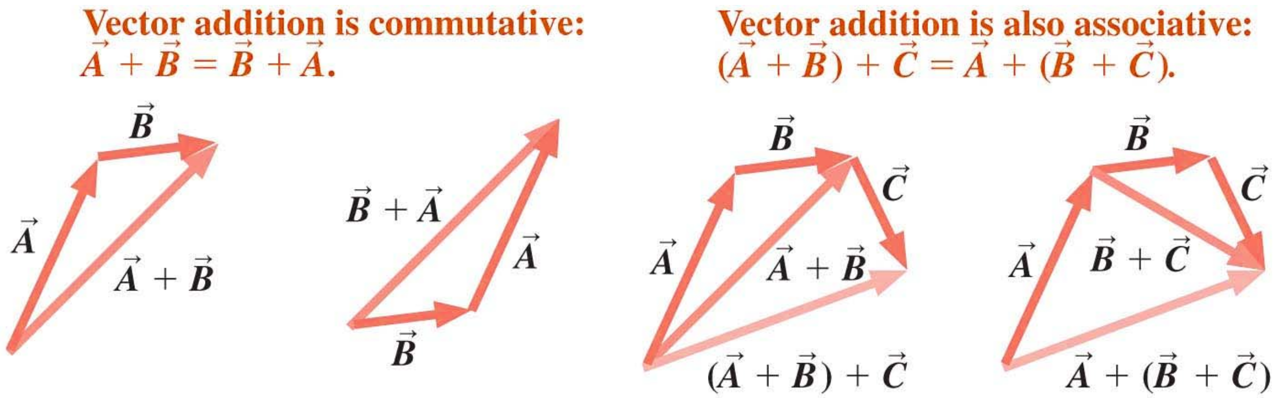
Vector arithmetic

- To multiply a vector by a scalar, multiply the vector's magnitude by the scalar.
  - For a positive scalar the direction is unchanged.
  - For a negative scalar the direction reverses.

- To subtract vectors, add the negative of the second vector to the first:

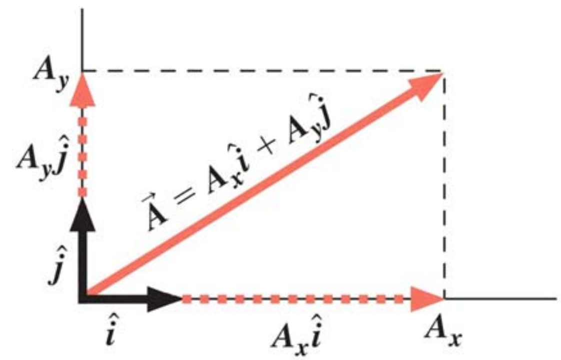
$$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

- Vector arithmetic is commutative and associative:

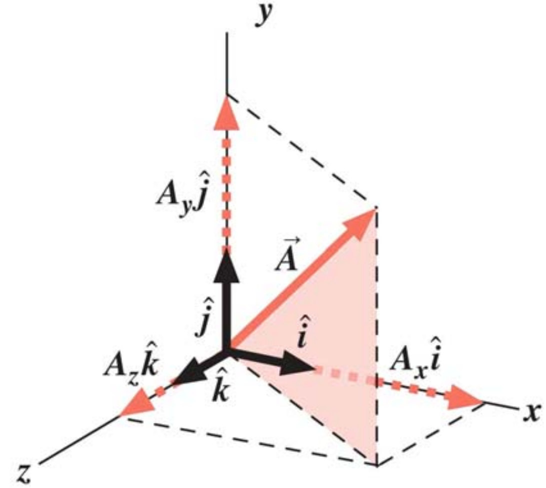


Unit vectors

- Unit vectors have magnitude 1, no units, and point along the coordinate axes.
  - They're used to specify direction in compact mathematical representations of vectors.
  - Unit vectors in the *x*, *y*, and *z* directions are designated $\hat{i}$, $\hat{j}$, and $\hat{k}$.



- Any vector in two dimensions can be written as a linear combination of $\hat{i}$ and $\hat{j}$.
- Any vector in three dimensions can be written as a linear combination of $\hat{i}$, $\hat{j}$, and $\hat{k}$.



Vector arithmetic with unit vectors

- To add vectors, add the individual components:
  - If $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$
  - then $\vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$

- To multiply by a scalar, distribute the scalar; that is, multiply the individual components by the scalar:
  - If $\vec{A} = A_x\hat{i} + A_y\hat{j}$
  - then $c\vec{A} = cA_x\hat{i} + cA_y\hat{j}$

Velocity and acceleration vectors

- Velocity is the rate of change of position.
  - The average velocity over a time interval Δt is the change in the position vector divided by the time.
    - Here dividing by Δt means multiplying by the scalar $1/\Delta t$:

$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t}$$

- Instantaneous velocity is the time derivative of position:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

- Acceleration is the rate of change of velocity:

Average: $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$ Instantaneous: $\vec{a} = \frac{d\vec{v}}{dt}$

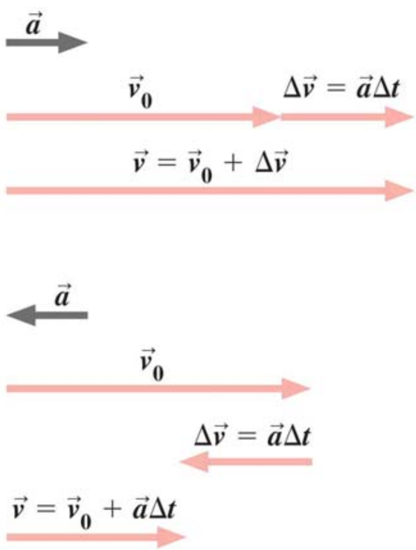
Velocity and acceleration in two dimensions

- An acceleration $\vec{a}$ acting for time Δt produces a velocity change $\Delta\vec{v} = \vec{a}\,\Delta t$.
  - The change adds *vectorially* to give the new velocity:

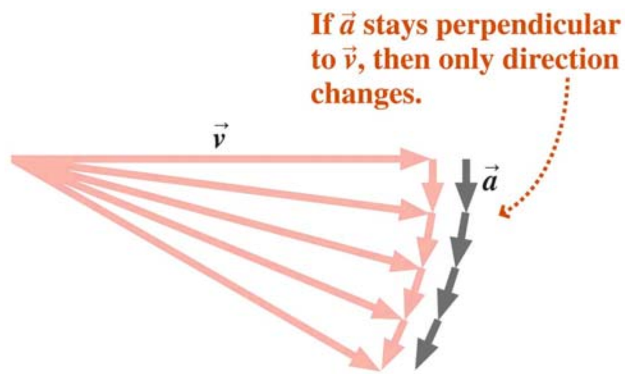
$$\vec{v} = \vec{v}_0 + \vec{a}\,\Delta t$$

- The new velocity depends on the magnitude of the acceleration as well as its direction:

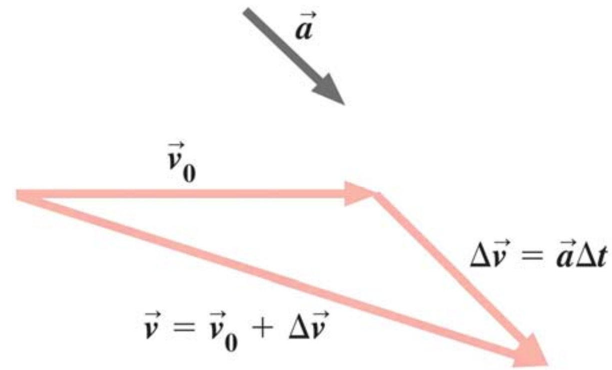
$\vec{a}$ and $\vec{v}$ colinear:
only speed changes



$\vec{a}$ and $\vec{v}$ perpendicular:
only direction changes



In general:
both speed and direction change



question

- An object is accelerating downward. Which of the following must be true?

A. The object is moving directly downward.

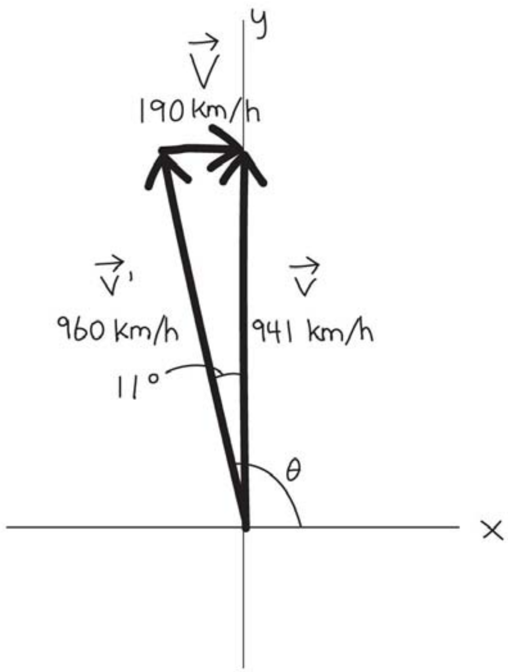
B. If the object's motion is instantaneously horizontal, it can't continue to be so.

C. The object cannot be moving in a straight line.

D. The object cannot be moving upward.

Relative motion

- An object moves with velocity $\vec{v}'$ relative to one frame of reference.
- That frame moves at $\vec{V}$ relative to a second reference frame.
- Then the velocity of the object relative to the second frame is $\vec{v} = \vec{v}' + \vec{V}$.
- Example:
  - A jetliner flies at 960 km/h relative to the air, heading northward. There's a wind blowing eastward at 190 km/h. In what direction should the plane fly?
  - The vector diagram identifies the quantities in the equation, and shows that the angle is 11°.



Constant acceleration

- With constant acceleration, the equations for one-dimensional motion apply independently in each direction.
  - The equations take a compact form in vector notation.
  - Each equation stands for two or three separate equations.

$$\vec{v} = \vec{v}_0 + \vec{a}\,t$$
$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

question

- An object is moving initially in the $+x$ direction. Which of the following accelerations, all acting for the same time interval, will cause the greatest change in its speed?

A. $10\hat{j}$ m/s^2

B. $2\hat{i} - 8\hat{j}$ m/s^2

C. $10\hat{i}$ m/s^2

D. $10\hat{i} + 5\hat{j}$ m/s^2