Recap: Position and displacement

- In one dimension, position can be described by a positive or negative number on a number line, also called a coordinate system.
  - Position zero, the origin of the coordinate system, is arbitrary and you’re free to choose it wherever it’s convenient.
- **Displacement** is change in position.
  - For motion along the $x$ direction, displacement is designated $\Delta x$:
    \[ \Delta x = x_2 - x_1 \]
    where $x_1$ and $x_2$ are the initial and final positions, respectively.
Recap: Velocity

- **Velocity** is the rate of change of position.
  - **Average velocity** over a time interval $\Delta t$ is defined as the displacement divided by the time:
    $$ v = \frac{\Delta x}{\Delta t} $$
  - **Instantaneous velocity** is the limit of the average velocity as the time interval becomes arbitrarily short:
    $$ v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} $$
    - In calculus, this limiting procedure defines the derivative $dx/dt$.
  - **Speed** is the magnitude of velocity.

- Velocity is the slope of the position-versus-time curve.

Recap: Using calculus to find derivatives

- In calculus, the derivative gives the result of the limiting procedure.
  - Derivatives of powers are straightforward:
    $$ \frac{d(bt^n)}{dt} = bnt^{n-1} $$
  - Other common derivatives include the trig functions:
    $$ \frac{d(\sin bt)}{dt} = b\cos bt $$
    $$ \frac{d(\cos bt)}{dt} = -b\sin bt $$
Recap: Acceleration

- **Acceleration** is the rate of change of velocity.
  - **Average velocity** over a time interval $\Delta t$ is defined as the change in velocity divided by the time:
    \[ \bar{v} = \frac{\Delta v}{\Delta t} \]
  - **Instantaneous acceleration** is the limit of the average acceleration as the time interval becomes arbitrarily short:
    \[ a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \]

Recap: Position, velocity, and acceleration

- Individual values of position, velocity, and acceleration aren’t related.
  - Instead, velocity depends on the rate of change of position.
  - Acceleration depends on the rate of change of velocity.
  - An object can be at position $x = 0$ and still be moving.
  - An object can have zero velocity and still be accelerating.

At the peak of its trajectory, a ball has zero velocity, but it’s still accelerating.
**Constant acceleration**

- When acceleration is constant, then position, velocity, acceleration, and time are related by
  
  - \[ v = v_0 + at \]
  - \[ x = x_0 + \frac{1}{2} (v_0 + v) t \]
  - \[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]
  - \[ v^2 = v_0^2 + 2a(x - x_0) \]

  where \( x_0 \) and \( v_0 \) are initial values at time \( t = 0 \), and \( x \) and \( v \) are the values at an arbitrary time \( t \).

- With constant acceleration
  - Velocity is a linear function of time
  - Position is a quadratic function of time

**question**

- A speeding motorist zooms past a stationary police car, which then heads after the speeder. The police car starts with zero velocity and is going at twice the car’s velocity when it catches up to the car. So at some intermediate instant the police car must be going at the same velocity as the speeding car. When is that instant?
  
  A. Closer to the time when the police car starts chasing
  
  B. Closer to the time when the police car catches the speeder
  
  C. Halfway between the times when the two cars coincide
The acceleration of gravity

- The acceleration of gravity at any point is exactly the same for all objects, regardless of mass.
- Near Earth’s surface, the value of the acceleration is essentially constant at \( g = 9.8 \text{ m/s}^2 \).
- Therefore the equations for constant acceleration apply:
  - In a coordinate system with \( y \) axis upward, they read
    \[
    v = v_0 - gt \\
    y = y_0 + \frac{1}{2}(v_0 + v)t \\
    y = y_0 + v_0t - \frac{1}{2}gt^2 \\
    v^2 = v_0^2 - 2g(y - y_0)
    \]

This strobe photo of a falling ball shows increasing spacing resulting from the acceleration of gravity.

Example: the acceleration of gravity

- A ball is thrown straight up at 7.3 m/s, leaving your hand 1.5 m above the ground. Find its maximum height and when it hits the floor.
  - At the maximum height the ball is instantaneously at rest (even though it’s still accelerating). Solving the last equation with \( v = 0 \) gives the maximum height:
    \[
    0 = v_0^2 - 2g(y - y_0)
    \]
    or
    \[
    y = y_0 + \frac{v_0^2}{2g} = 1.5 \text{ m} + \frac{(7.3 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 4.2 \text{ m}
    \]
  - Setting \( y = 0 \) in the third equation gives a quadratic in time; the result is the two values for the time when the ball is on the floor:
    \( t = -0.18 \text{ s} \) and \( t = 1.7 \text{ s} \)
  - The first answer tells when the ball would have been on the floor if it had always been on this trajectory; the second is the answer we want.
question

• Standing on a roof, you simultaneously throw one ball straight up and drop another from rest. Which hits the ground moving faster?

A. The ball dropped from rest
B. The ball thrown straight up

Summary

• Position, velocity, and acceleration are the fundamental quantities describing motion.
  • Velocity is the rate of change of position.
  • Acceleration is the rate of change of velocity.

• When acceleration is constant, simple equations relate position, velocity, acceleration, and time.
  • An important case is the acceleration due to gravity near Earth’s surface.
  • The magnitude of the gravitational acceleration is \( g = 9.8 \text{ m/s}^2 \).

\[
\begin{align*}
v &= v_0 + at \\
x &= x_0 + \frac{1}{2}(v_0 + v)t \\
x &= x_0 + v_0t + \frac{1}{2}at^2 \\
v^2 &= v_0^2 + 2a(x - x_0)
\end{align*}
\]