CLASSICAL MECHANICS/ ELECTRICITY AND MAGNETISM

Preliminary Examination

August 25, 2010

9:00 - 15:00 in P-121

Answer a total of **SIX** questions, choosing **THREE** from each section. If you turn in excess solutions, the ones to be graded will be picked at random.

Each answer must be presented **separately** in an answer book or individual sheets of paper. Make sure you clearly indicate who you are, and the problem you are answering on each book/sheet of paper. Double-check that you include everything you want graded, and nothing else.

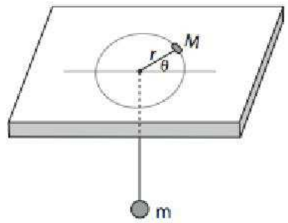

Figure 1: For problem **A2**.

SECTION A - Classical Mechanics

A1. Calculate the moment of inertia of a (non-uniform) solid sphere of mass M and radius R when it is rotating about a special z-axis through its center. Assume that the density ρ of the sphere varies as

$$\rho(\mathbf{r}) = c + kz$$

with the origin of the coordinate system being at the center of the sphere; here c and k are constants such that $0 \leq k < c/R$. Express your answer in terms of the mass M and other relevant constants. Compare your answer with the uniform case where $k = 0$ and explain your results.

A2. Two masses M and m are connected by a massless string of length l that passes through a hole in a horizontal table. Assume gravitational acceleration g acting vertically downwards, perpendicular to the table. Mass M is moving on the table along a circle of radius r at a constant angular velocity ω. Ignore friction here.

(a) What are the equations of motion of the masses?

(b) Derive an expression for r as a function of the angular velocity ω.

(c) Now assume that both r and ω are allowed to be time dependent. Obtain a differential equation for r (with no angular terms). **Do not try to solve it.**

A3. Triplets are 22 years old. One triplet stays on Earth, while the other two triplets make round-trip rocket journeys to two stars equally distance from the earth in opposite directions at the speed of 0.8 c, where c is the speed of light in a vacuum. The two traveling triplets return to Earth at age 34.

(a) How old is the triplet who stayed at home when the siblings arrive back?

(b) How far away are the stars in light years?

(c) How fast do the two rockets move relative to each other on their trips out and back to the stars?

(Assume that the time required to accelerate and decelerate the rocket can be neglected.)

A4. Consider a rigid body rotating about a fixed hinge O in an inertial frame F. Write down Euler's equations of motion identifying your symbols carefully.

Using Euler's equations show that the rate of change of kinetic energy, dT/dt, is given by $\vec{\omega} \cdot \vec{\tau}$, where $\vec{\omega}$ and $\vec{\tau}$ represent the angular velocity and the torque about O, respectively.

If a rigid, uniform cuboid is set rotating about a principal axis through a hinge at a corner and it is acted by no forces other than the ones passing through the hinge, show that the angular velocity is a constant vector in a non-rotating inertial frame.

SECTION B - Electricity and Magnetism

B1. The problem of the determination of a variable electromagnetic field in free space for given charge and current distributions, $\rho(\mathbf{r}, t)$ and $\mathbf{j}(\mathbf{r}, t)$, may be solved by evaluating the retarded potentials $\phi(\mathbf{r}, t)$ and $\mathbf{A}(\mathbf{r}, t)$ where these potentials satisfy d'Alembert equations and are related by the Lorentz condition.

Write down expressions for the above potentials in terms of their sources, identify the Lorentz condition and obtain the vector potential at large distances r far away from the sources in the so-called radiation zone.

Find the electric and magnetic fields in the radiation zone due to an oscillating electric dipole of the form

$$\mathbf{p} = \mathbf{p_0} \cos(\omega t)$$

where $\mathbf{p}_0$ is a constant (i.e., time independent) vector.

B2. Show that the attractive force between a charge Q and a neutral atom is proportional to Q^2/R^5 where R is the separation between the atom and the charge. (Hint: The atom can have an induced dipole moment.)

B3. Two long cylindrical conductors of radius a are parallel and separated by a distance d which is large compared to a. Show that the capacitance per unit length is given approximately by

$$C \propto (4\ln(d/a))^{-1}.$$

B4. Earth's magnetic field is sustained from motions of an electrically conducting fluid in its outer core, rotating in cylindrically shaped volume whose axis is quasi-parallel to the Earth's rotational axis.

(a) Using Ohm's law for the current density of fluid conductor in motion,

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}),$$

and Maxwell's equations show that the Earth's magnetic field will decay to zero when the velocity of the fluid goes to zero.

(b) Write an approximate formula for the time constant of decay (time for the magnitude of the field to be e^{-1} of its starting value) containing electromagnetic properties of the core and a length scale for the diameter of the cylindrical fluid motion.

Hint: $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$.