CLASSICAL MECHANICS

Preliminary Examination

January 14, 2013

9:00 - 12:00 in P-121

Answer a total of **THREE** questions. If you turn in excess solutions, the ones to be graded will be picked at random.

Each answer must be presented **separately** in an answer book or on individual sheets of paper stapled together. Make sure you clearly indicate who you are, and the problem you are answering. Double-check that you include everything you want graded, and nothing else.

Classical Mechanics

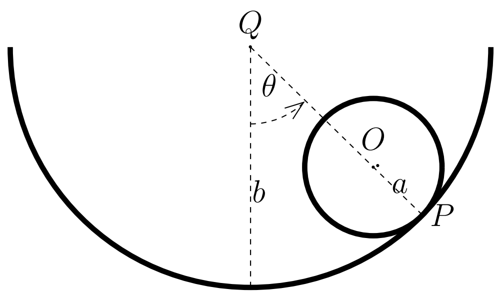

Fig. 1

1. Consider a small sphere of radius a and mass m <u>rolling</u> (up or down) inside a fixed, hemispherical bowl of radius b with $b > 2a$. Let θ be the angle between the line (QO) joining the centers of the spheres and the downward vertical. Assume that this line, as it moves, spans the vertical plane as shown in Fig. 1.

(a) Write down a Lagrangian for the case when the small sphere is allowed to roll freely inside the bowl in the vertical plane spanned by the line (QOP) containing the centers. Identify the number of degrees of freedom noting that the angular speed of the small sphere is not necessarily equal to $\dot{\theta}$.

(b) Locate an equilibrium position for the small sphere and determine whether the equilibrium is stable or not.

(c) Using an equation of motion or otherwise, show that the rolling frictional force f_R acting on the small sphere at any time is

$$f_R = \frac{2mg\sin\theta}{7}.$$

Does it do work? Explain.

2. Consider the setup in Fig. 2, consisting of <u>three masses</u>, m, m, and M, connected by four weightless rods, each of length l. The whole setup is fixed to a rotating rod and every rod can move without deformation. However, note that the relative angles between the rods can change. The whole system rotates around the vertical (symmetry) axis with a constant angular speed ω.

(a) How many degrees of freedom does this problem have? Write down the Lagrangian and derive the Lagrange's equation(s) of motion.

(b) Find the equilibrium. At what angular speeds ω is the stable equilibrium nontrivial, i.e., not all rods hang straight down?

(c) Describe small oscillations around the nontrivial equilibrium point.

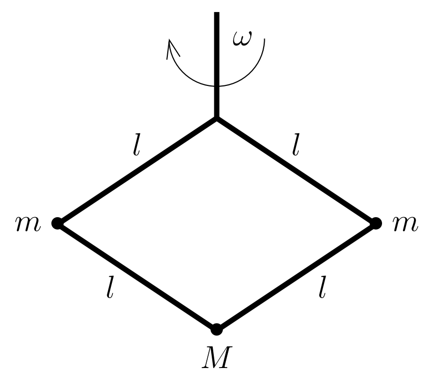

Fig. 2

3. Consider a central potential

$$V(r) = \frac{A}{r^2} - \frac{B}{r}$$

with (real) positive and constant values for A and B. Assume that the orbital angular momentum, which is a constant of motion here, is fixed at l.

(a) What is the equivalent one-dimensional problem? Plot the relevant effective potential (V_{eff}) as a function of r.

(b) In your plot, identify two energies E_1 and E_0 that would give rise to elliptical and circular orbits respectively. Explain your reasoning and determine the radius r_0 of the (stable) circular orbit.

(c) Expanding the effective potential $V_{eff}(r)$ up to second order in δr around the radius r_0, find and solve an equation of motion for small variations δr in radius.

(d) Provide an intuitive argument as to whether a small increase in the orbital radius in part (c) should increase or decrease the angular speed.

4. After suitable scaling, the Hamiltonian for an isotropic harmonic oscillator in 2-dimensions may be written as

$$\frac{1}{2}\sum_{i=1}^{2}({p_i}^2 + {x_i}^2).$$

(a) Write and solve Hamilton's equations of motion.

(b) Show using Poisson brackets that the quantities

$$A_{ij} = \frac{1}{2}(p_i p_j + x_i x_j),\ \ i,j = 1,2$$

are constants of motion.

(c) Give a physical interpretation for A_{ij}.