Quantum Mechanics and Bell's Inequalities

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Abstract

Santos argues that, if one interprets probabilities as ratios of detected events to copies of the physical system initially prepared, the quantum mechanical predictions for the classic tests of Bell's inequalities do not violate the inequalities. Furthermore, he suggests that quantum mechanical states which do violate the inequalities are not physically realizable. We discuss a physically realizable experiment, meeting his requirements, where quantum mechanics does violate the inequalities.

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Santos [1–4] claims it has not been shown that quantum mechanics violates the Bell inequalities [5]. He argues that quantum probabilities should correspond to the number of detected events divided by the number of copies of the physical system initially prepared, and uses this proposition to analyze idealized versions of the classic tests [6,7] of Bell's inequalities. For example, Santos considers an experiment where the correlated photons from a $J = 0 \rightarrow J = 1 \rightarrow J = 0$ atomic cascade are recorded with perfect analyzers and detectors, and identifies the probabilities that appear in Bell's inequality [5]

$$p(a_1) + p(b_2) \geq p(a_1, b_2) + p(a_1, d_2) + p(c_1, b_2) - p(c_1, d_2) \tag{1}$$

with the ratios of the detected events to the number of atomic states initially prepared. The isotropic nature of the single-photon angular distribution causes the single probabilities (which appear on the left-hand side of the inequality) to contain a factor $\Omega/4\pi$ and the joint probabilities (which appear on the right-hand side of the inequality) to contain a factor $a(\gamma)(\Omega/4\pi)^2$, where γ is the opening angle between the two emitted photons. The quantum mechanical prediction for the angular correlation function $a(\gamma)$ is

$$a(\gamma) = \frac{3}{4}(1 + \cos^2 \gamma) \ . \tag{2}$$

For small detector solid-angles $\Omega/4\pi \ll 1$, the quantum mechanical predictions for the probabilities so defined do not violate Eq. 1. Santos showed that if one attempts to avoid this problem by making $\Omega/4\pi$ larger, the finite angular acceptance attenuates the quantum-mechanical two-photon polarization-correlation function in such a way that Bell's inequality remains satisfied [8].

The angular acceptance problem can be avoided by studying the annihilation of a singlet state, such as the π^0 or para-positronium [9]. The two-photon annihilation angular correlation is no longer given by Eq. 2 but becomes

$$a(\gamma) = \delta(\gamma) \ . \tag{3}$$

If the initial system is centered between identical detectors A and B, and if one photon enters detector A, the other must enter detector B. Thus both the *single* and *joint* probabilities contain the same solid-angle factor $\Omega/2\pi$ [10] which allows the quantum mechanical prediction to violate the Bell inequalities. This violation is readily calculated from the two-photon $J^\pi = 0^-$ wave function expressed in a linear polarization basis

$$|\Psi(\nu_1, \nu_2)\rangle = \frac{|\hat{x}_1, \hat{y}_2\rangle - |\hat{y}_1, \hat{x}_2\rangle}{\sqrt{2}} \ . \tag{4}$$

Suppressing the common factor of $\Omega/2\pi$ that, according to Santos' prescription, should normalize all probabilities, the quantum-mechanical single and joint probabilities are

$$p(\theta) = \frac{1}{2} \ , \qquad p(\theta_1, \theta_2) = \frac{\sin^2(\theta_1 - \theta_2)}{2} \ , \tag{5}$$

where θ_1 and θ_2 denote the orientations of polarizers 1 and 2. If a, b, c, and d refer to $\theta = 0$, $3\pi/8$, $3\pi/4$, and $5\pi/8$ respectively, then

$$p(a_1, b_2) + p(a_1, d_2) + p(c_1, b_2) - p(c_1, d_2) \approx 1.207 \ , \tag{6}$$

which exceeds the upper limit of unity allowed by Bell's inequality. However, because the efficiencies of detectors and polarization analyzers for π^0 or para-positronium decay cannot be as high as they are in atomic experiments, Santos and others [11] argue that quantum mechanics does not actually violate Bell's inequality in annihilation experiments. Once the maximum linear-polarization analyzing power allowed by quantum mechanics (*e.g.* the Klein-Nishina cross section for a Compton polarimeter) is included, the quantum mechanical probabilities for the *detected* events (Compton scattered photons) do not violate the inequality. A survey of the experiments performed to-date points to a trade-off between analyzer and detector efficiencies; those experiments which implemented nearly-ideal polarizers were limited in the efficiency of the detectors, and *vice-versa.*

These arguments are the basis for Santos' claim that no existing experiment rules out local hidden-variable theories. In fact, Santos goes farther and suggests [2,4] that those quantum mechanical states that do violate the inequalities are not physically realizable. He suggests, for example, that the requirements of high detection efficiency and high polarization analyzing power are not simultaneously realizable because one relies on the particle properties of the detected quantum and the other on its wave properties. Santos' arguments are clearly stated and raise an interesting question of principle. Are his observations indeed signs that something 'protects' hidden variables in a manner analogous to the way the uncertainty principle 'protects' quantum mechanics from attempts to show it is contradictory? Or do they instead reflect properties of particular physical systems and say nothing profound about quantum mechanics? In this Letter, we discuss a physically realizable (but extraordinarily difficult) experiment for which the quantum mechanical predictions as interpreted by Santos do violate the inequalities.

Consider an experiment involving two monochromatic counter-propagating beams of very-cold neutrons (we call a neutron very-cold if its DeBroglie wavelength is so long that it cannot Bragg scatter). At these very low energies, neutrons that scatter are in an s state. Hence the two identical fermions that result from the scattering must have $S = 0$ [12] and a spin wave function

$$|\Psi(n_1, n_2)\rangle = \frac{|\hat{z}_1, -\hat{z}_2\rangle - |-\hat{z}_1, \hat{z}_2\rangle}{\sqrt{2}} , \tag{7}$$

where $\hat{z}$ denotes an arbitrary spin quantization axis. (Note that finite beam and detector solid angles do not have any appreciable effect on this argument because p and d-wave scattering is utterly negligible for very-cold neutrons.) We therefore have a correlated state of back-to-back fermions analogous to the back-to-back photons resulting from para-positronium decay, but now the quantum mechanical single and joint probabilities are

$$p(\theta) = \frac{1}{2} , \qquad p(\theta_1, \theta_2) = \frac{1}{2} \sin^2 \left(\frac{\theta_1 - \theta_2}{2} \right) , \tag{8}$$

so that the right hand side of Eq. 1 is approximately 1.207 when a, b, c, and d refer to $\theta = 0$, $3\pi/4$, $3\pi/2$, and $5\pi/4$ respectively. However, in contrast to the annihilation example, one now has analyzers (scattering from magnetized multi-layer foils) and detectors (^{6}Li-doped glass scintillators that detect neutrons *via* the $^6\mathrm{Li}(n, t)\alpha$ reaction) with analyzing powers and efficiencies *very close to unity* (see ref. [13,14] for discussions of such devices and their performance) so that the actual counting rates can violate the inequality.

For concreteness, we assume neutrons thermalized in liquid deuterium and selected to have velocities $v \approx 800$ m/s near the peak of the Maxwell-Boltzmann distribution, and consider 90° scattering. If one of the scattered neutrons enters counter A the other neutron should have a high probability of entering counter B. This restricts the allowable velocity-spread, Δv_z, of the neutron beams and dictates that the intersection region be collimated [16] to a length d that is small compared to the counter radius a (see Fig. 1). (Note that the detector solid angles can be large because the polarization correlation is not attenuated by finite solid angles.) Values of $\Delta v_z/v \approx 0.01$ and $d/a \approx 0.01$ are more than sufficient.

To test *relativistic locality*, the detected pairs of events must have a space-like separation. This places a strong requirement [4] on the phase-space of the neutron beams in the coordinate, x, along the line joining the two counters. If the x phase space is limited only by the uncertainty principle (although this corresponds to a beam emittance of 10^{-4} mm mr, we remind the reader that we are discussing questions of principle) the detectors must be separated from the beam by a distance

$$R > \frac{\rho^2 \hbar c^2}{4 m v_x^3} \left(1 + \sqrt{1 + 8 \left(\frac{m v_x^2 \lambda}{\rho \hbar c} \right)^2} \right) \tag{9}$$

where m and v_x are the neutron mass and velocity, λ is the mean neutron absorption length in the detectors [15], and ρ is the time required for light to travel from one detector to the other measured in units of the coincidence time resolution σ_t. We require $\rho = 2$ so that at least 95% of the coincidence events fall within the 'space-like gate'. Commercial suppliers of ^{6}Li glass scintillators typically dope the material so that $\lambda = 0.17$ mm for 800 m/s neutrons. Therefore the requirements of Eq. 9 are satisfied for $R = 100$ m, which is not impossible; a magnetic field of 0.1 mGauss would precess the neutron spins by about 230 mr, and gravity would cause the neutrons to fall by $\sim$8 cm over this path length. (A higher doping level and concomitant decrease in λ would shorten R considerably.)

How good must the analyzer/detectors be to observe a violation of Bell's inequality? We specify the performance of an analyzer/detector in terms of its efficiency ϵ_p (ϵ_a) for counting neutrons with spins parallel (anti-parallel) to the 'easy axis' and define $\epsilon = \epsilon_p + \epsilon_a$ and

$A = (\epsilon_p - \epsilon_a)/\epsilon$. The observed counting rates will violate Eq. 1 for the angles given below Eq. 8 if

$$f\epsilon(1 + \sqrt{2}A^2) > 2 \ , \tag{10}$$

where $f = 0.95$ is the fraction of coincident events lying within the 'space-like gate'. For an achievable value of $\epsilon = 0.97$, Eq. 10 is satisfied for $A > 0.91$. Values better than this have been obtained.

The difficulty of this experiment raises the question whether there is a less extravagant way to test Santo's conjecture. Several tests [17–20] of Bell's inequality have been reported using parametric down-conversion, which provides a convenient source of photon pairs in the visible region with good spatial and energy correlations. Because there are no detectors in the visible region with efficiencies $\geq 85\%$ and good time resolution, the efficiency loophole cannot yet be closed by such experiments [21]. The most competitive detector appears to be the silicon avalanche diode, but considerable progress is needed to close the gap between the current state-of-the-art and the conditions that we describe in our cold neutron experiment. It should be noted that the couplers, filters and collimators often used in optics experiments would in some cases preclude a direct violation of Bell's inequality even for ideal detectors and analyzers.

We have presented a physically realizable experiment for which the quantum mechanical probabilities *as interpreted by Santos* violate the Bell inequalities, and conclude that the interesting observations of Santos do not reflect an underlying property of quantum mechanics but are instead features of the particular systems he considered. Although we have not shown that *Nature* violates Bell's inequalities, we have confirmed that the inequalities provide a physically realizable (but admittedly very difficult) way to distinguish local hidden variable theories from quantum mechanics.

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[12] Although neutrons scattered by the *strong* interaction are in an s-state and have $S = 0$, *magnetic* scattering always produces neutrons in an $S = 1$ state. However, magnetic

scattering can be neglected; for unpolarized beams the magnetic scattering cross-section is isotropic and has a magnitude $\sigma = (\pi/2)(m_n)^2(\mu_n/\hbar)^4$ that is roughly 10^{-9} of the strong cross-section.

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[15] For simplicity, this expression approximates as Gaussians the exponential distributions of neutron interaction points in the detectors. Numerical calculations verify the reasonableness of our approximation.

[16] It has been conjectured [4] that, because the Dirac Hamiltonian does not commute with the orbital and spin angular momentum operators, collimation can affect the spin states of s-wave fermions. Even if this is correct, it is of no practical importance in our case. The quantum mechanical violation of Bell's inequality is of order 10^{-1}, while our neutrons have $v/c \sim 10^{-6}$.

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[21] Although CCD's have been demonstrated with efficiencies close to this value, CCD's are not appropriate for such experiments because they are integrating devices that cannot provide an accurate time measurement.

FIGURES

FIG. 1. Schematic layout of the neutron scattering experiment.