



Mie Scattering

Physics 259 - DS Hamilton 2005

This worksheet investigates the modified Rayleigh-Debye approximation for Mie scattering as discussed by R.M. Drake and J.E. Gordon, Am. J. Phys. **53** 955 (1985). The calculation is done for light polarized perpendicular to the scattering plane. The results for $ka=10$ are compared to a full Mie scattering calculation

$$m := 1.2$$

relative index of refraction

$$\lambda := 0.6328$$

wavelength of the light in microns (μ)

$$ka := 10$$

$$k=2\pi/\lambda$$

we can choose to fix either ka or a , I chose ka .

$$a := ka \cdot \frac{\lambda}{2 \cdot \pi}$$

$$2a = 2.014$$

$2a$ is the sphere diameter

$$\text{cosd}(\theta) := \cos(\theta \cdot \text{deg})$$

I want to do the calculation with the scattering angle in degrees and not radians

$$x(\theta) := ka \cdot \left(1 + m^2 - 2 \cdot m \cdot \text{cosd}(\theta)\right)^{0.5}$$

a modification to the usual R-D approximation

$$J1(z) := \frac{\sin(z) - z \cdot \cos(z)}{z^2}$$

This defines the first-order spherical Bessel function

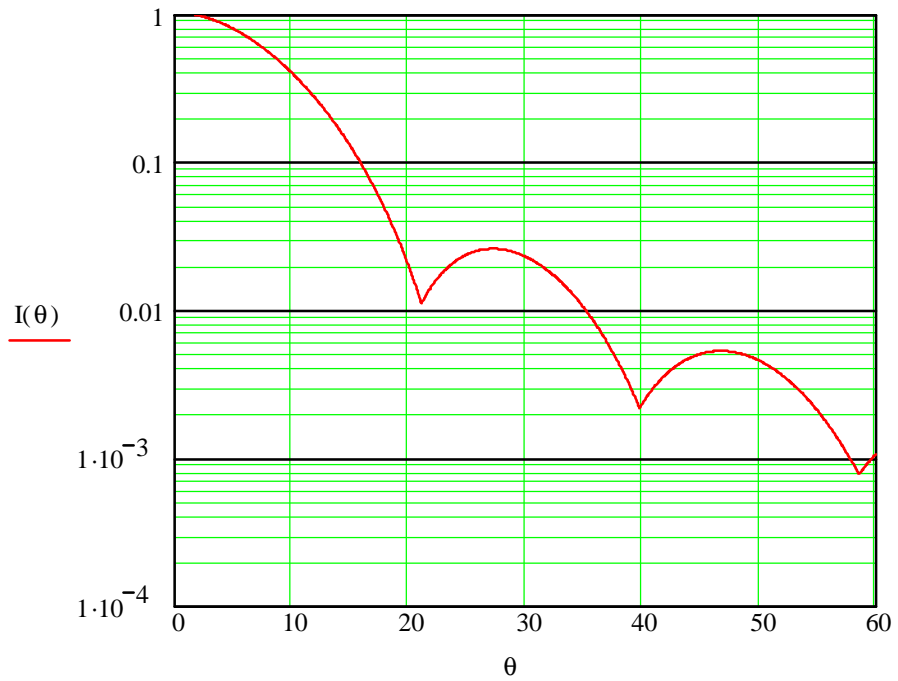
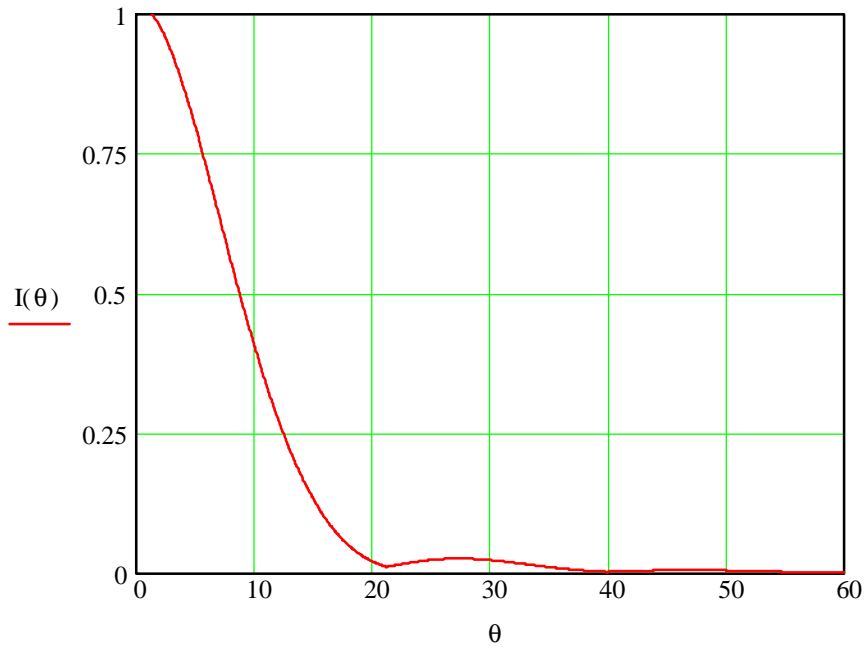
$$\gamma(\theta) := \frac{\text{sign}(J1(x(\theta)))}{x(\theta)^{1.5}}$$

another improvement to the R-D approximation

$$I(\theta) := \left(\frac{3 J1(x(\theta))}{x(\theta)} + \gamma(\theta) \right)^2$$

The scattering efficiency as a function of scattering angle. The forward direction is $\theta=0^\circ$ and $I(0^\circ)=1.0$.

Here are the graphs of the intensity of the scattered light as a function of scattering angle. On the semi-log graph you can clearly see the local minima at 22° and 40° .



We next compare the modified R-D approximation to the full Mie scattering calculation using the results on the web at <http://omic.nyu.edu>. Values of $\lambda=0.6328\mu$ and a diameter of 2.014μ give us $ka=10$.

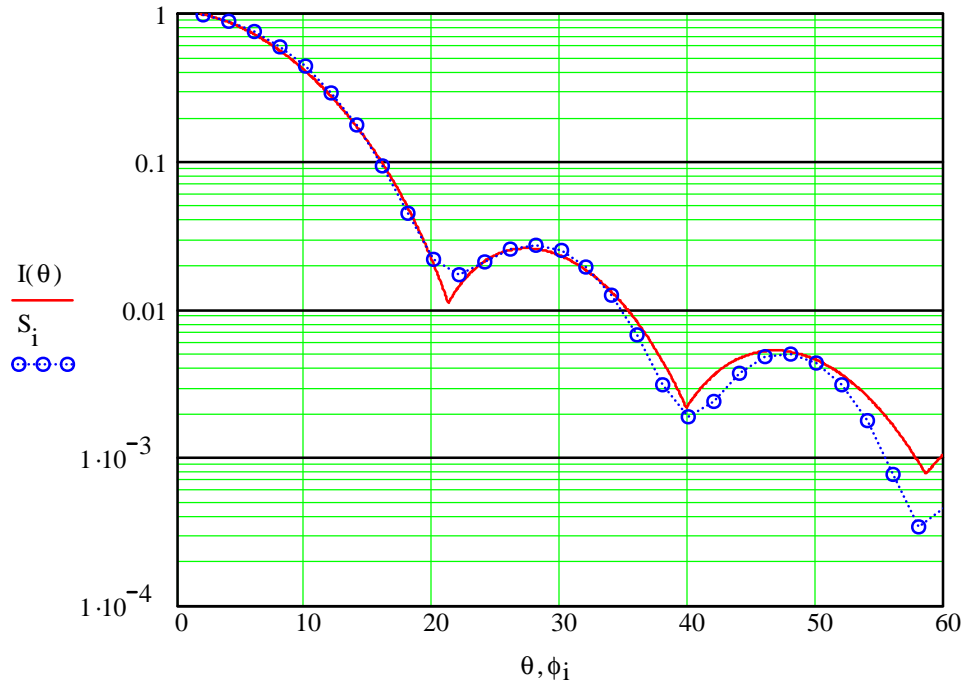
$i := 1..60$

$S :=$

| |
|-------|
| 1 |
| 0.969 |
| 0.881 |
| 0.749 |
| 0.593 |
| 0.434 |

$\phi :=$

| |
|----|
| 0 |
| 2 |
| 4 |
| 6 |
| 8 |
| 10 |



The R-D approximations seem to be best for the smaller scattering angles and for $ka < 30$.