This worksheet investigates the modified Rayleigh-Debye approximation for Mie scattering as discussed by R.M. Drake and J.E. Gordon, Am. J. Phys. 53 955 (1985). The calculation is done for light polarized perpendicular to the scattering plane. The results for $ka=10$ are compared to a full Mie scattering calculation.

\[ m := 1.2 \]  
\[ \lambda := 0.6328 \]  
\[ ka := 10 \quad k=2\pi\lambda \]  
\[ a := ka \frac{\lambda}{2\pi} \quad 2a = 2.014 \]

\[ \cosd(\theta) := \cos(\theta\cdot\text{deg}) \]

\[ x(\theta) := ka \left(1 + m^2 - 2m \cdot \cosd(\theta)\right)^{0.5} \]

\[ J_1(z) := \frac{\sin(z) - z \cdot \cos(z)}{z^2} \]

\[ \gamma(\theta) := \frac{\text{sign}(J_1(x(\theta)))}{x(\theta)^{1.5}} \]

\[ I(\theta) := \left(\frac{3 J_1(x(\theta))}{x(\theta)} + \gamma(\theta)\right)^2 \]

The scattering efficiency as a function of scattering angle. The forward direction is $\theta=0^\circ$ and $I(0^\circ)=1.0$. 

The calculation was done with $ka=10$ and $a=2.014$, and we can choose to fix either $ka$ or $a$, I chose $ka$. 

2a is the sphere diameter.
Here are the graphs of the intensity of the scattered light as a function of scattering angle. On the semi-log graph you can clearly see the local minima at 22° and 40°.
We next compare the modified R-D approximation to the full Mie scattering calculation using the results on the web at http://omic.ogi.edu. Values of $\lambda = 0.6328 \mu$ and a diameter of $2.014 \mu$ give us $ka = 10$.

The R-D approximations seem to be best for the smaller scattering angles and for $ka < 30$. 

<table>
<thead>
<tr>
<th>$i$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_i$</td>
<td>1</td>
<td>0.969</td>
<td>0.881</td>
<td>0.749</td>
<td>0.593</td>
<td>0.434</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

The graph shows the scattering intensity $I(\theta)$ normalized by $S_1$ as a function of scattering angle $\theta, \phi_i$. The R-D approximations are indicated by the blue circles, while the full Mie calculation is shown by the red line.