Linear Regression I

Physics 258 - DS Hamilton 2004

This worksheet demonstates the Mathcad functions "slope" and "intercept" that are used to fit data to a straight line.

The data for this problem is from Bevington, page 97. The potential difference V has been measured as a function of position x along a current-carrying nickel-silver wire.

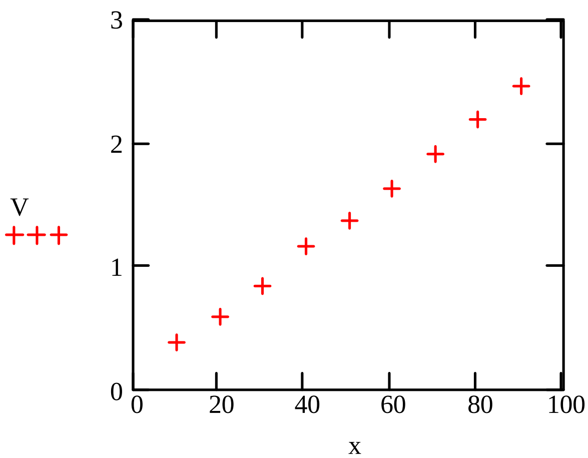
$$x := \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \\ 70 \\ 80 \\ 90 \end{bmatrix} \qquad V := \begin{bmatrix} 0.37 \\ 0.58 \\ 0.83 \\ 1.15 \\ 1.36 \\ 1.62 \\ 1.90 \\ 2.18 \\ 2.45 \end{bmatrix}$$

This data could also be entered into the worksheet as a table (Insert-Component-Input Table) or read in from a file. Here I used the Matrix pallet to generate two column vectors.

$n := \mathrm{rows}(x)$ $\qquad$ $n = 9$

This is just the number of data points.

Always plot your data before you try and fit it to some analytical function. You need to "see" what your data looks like.



Type: V@x <enter> to generate a quick plot of the data. This can often suggest a specific relationship (linear, quadratic, exponential etc) that will make a reasonable model to describe the data.

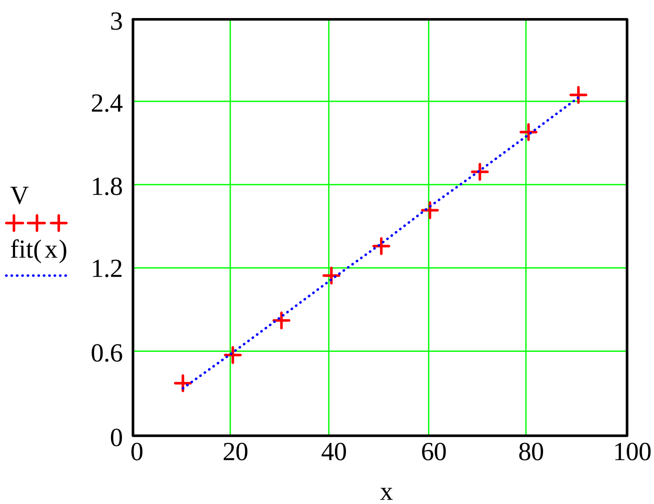
From the data it looks like a linear model with a slope m and an intercept b would be a good way to model the data. We will use the Mathcad built-in functions "intercept" and "slope".

$$b := \text{intercept}(x, V) \qquad b = 7.139 \cdot 10^{-2}$$

$$m := \text{slope}(x, V) \qquad m = 2.622 \cdot 10^{-2}$$

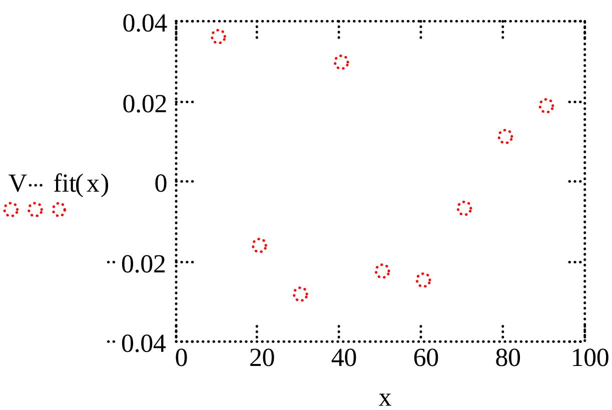
$$\text{fit}(x) := b + m \cdot x$$

This is our linear fit to the data.



A plot of the data V(x) and the linear fit.

Next we want to consider the quality of the fit. By plotting the residuals, (the difference between the data and the fit), we can look for any systematic errors or unexpected trends in the data.



type V-fit(X)@X

Some numerical values to asses the quality of the fit are:

$s_y := \sqrt{\dfrac{\sum (V - \mathrm{fit}(x))^2}{n-2}}$	$s_y = 2.6387 \cdot 10^{-2}$	Taylor Eq. 8.15
Standard error	$\mathrm{stderr}(x, V) = 2.6387 \cdot 10^{-2}$	just checking the built-in function
Correlation coefficient = R	$\mathrm{corr}(x, V) = 0.9994$	Taylor § 9.3 & 9.4
R^2	$\mathrm{corr}(x, V)^2 = 0.999$	