Error Propagation Using Mathcad

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Mathcad provides a powerful platform to allow you to do not only calculations, but also error propagation. Using the symbolic calculus functions, you can use the general error propagation formulas for any function.

The symbol ☞ indicates that you are being asked to perform some task. You should place your answer to the right of this symbol.

I. Symbolic Derivatives in Mathcad

Given a mathematical function, such as f(x,L) = exp(-x/L), mathcad can be used to calculate the partial derivatives with respect to any variable. This in turn will be used for calculating error propagation.

First, define the function f(x,L) by typing: `f(x,L) : x*L` or: `exp(-x/L)`

☞ $f(x,L) := \exp\left(\frac{-x}{L}\right)$

Now, make sure the calculus and symbolic toolbars are shown by selecting View -> Toolbars on the main menu and make sure that calculus and symbolic are highlighted.

To take the partial derivative of f(x,L) with respect to L, first press the d/dx button on the calculus toolbar. It should display on the screen

$$\frac{d}{d\blacksquare}\blacksquare$$

In the lower block, enter L and in the upper block enter f(x,L), so the result is $\frac{d}{dL}f(x,L)$

☞ $\frac{d}{dL}f(x,L) \rightarrow \frac{x}{L^2}\cdot\exp\left(\frac{-x}{L}\right)$

If you just press the enter key, nothing will be displayed. To display the result of a symbolic evaluation, on the symbolic equation menu, highlight the entire equation and press the right arrow key and press enter. It should display the derivative of the equation as shown below.

$$\frac{d}{dL}f(x,L) \rightarrow \frac{x}{L^2}\cdot\exp\left(\frac{-x}{L}\right)$$

Now, in your equation, we can verify that the equation is "live" by going to the definition of f(x,L) at the start of your worksheet and changing the definition of f(x,L) to, for example, f(x,L) = x/L, and the derivative will change on your screen.

Now we are in a position to evaluate the function and its derivatives for some numeric values. In the space below, enter x := 2 cm and L := 4 cm. Use a relatively simple function above (such as f(x,L) = x*y) and evaluate the function f for these values by typing f(x,L)=

☞

$x := 2\ \text{cm}$ $\quad$ $dx := .2\ \text{cm}$

$L := 4\ \text{cm}$ $\quad$ $dL := .4\ \text{cm}$

$\text{Result} := f(x, L)$

$\text{Result} = 0.607$

Verify that the value and units are both correct.

Finally, you can do your error propagation by using the calculus rule for error propagation (see one of our standard references). Define variables dx := 0.4 cm and dL := 0.2 cm. Then the error equation would be

$$\text{err}_x := \frac{d}{dx}(f(x, L)\cdot dx) \qquad \text{err}_L := \frac{d}{dL}(f(x, L)\cdot dL)$$

$$\text{Error} := \sqrt{(\text{err}_x)^2 + (\text{err}_L)^2}$$

$\text{Result} = 0.607$ $\quad$ $\text{Error} = 0.043$ $\quad$ $\left|\frac{\text{Error}}{\text{Result}}\right|\cdot 100\ \% = 7.071 \cdot \%$

The relative effect for each of the error terms would be given by

$$\left|\frac{\text{err}_x}{\text{Result}}\right|\cdot 100\ \% = 5 \cdot \%$$

$$\left|\frac{\text{err}_L}{\text{Result}}\right|\cdot 100\ \% = 5 \cdot \%$$

Adapted from the original web posting of Professor Tom Huber, Gustavus Adolphus College