Parallel LC Circuit

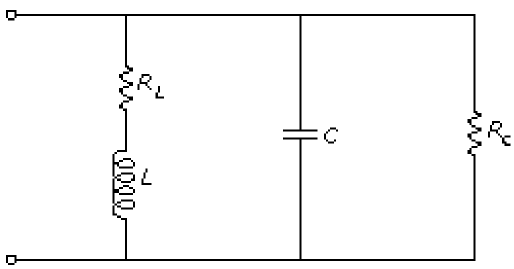
PHYSICS 258/259 D.S. Hamilton

Introduction

A parallel LC "tank" circuit is common in communications circuits. They are used in both oscillators and filters. In reality, there is a series resistance associated with the inductor and a parallel resistance associated with the capacitor. These are caused by the resistance of the wire used to wind the coil and the conductance of the dielectric used in the capacitor. This worksheet investigates the parallel LC circuit including modeling of these resistances.

Problem

Obtain the impedance $\mathbf{Z}(\omega)$ for the circuit shown below over the range $0 < \omega < \infty$. Plot the magnitude of $\mathbf{Z}$ and the phase angle as a function of ω on a log scale.



The figure is a parallel LC circuit with R_L in series with L and R_C in parallel with C.

Parameters

$L := 22.0\cdot\text{mH}$ $R_L := 15\cdot\Omega$

$C := 0.015\cdot\mu\text{F}$ $R_C := 100\cdot\text{M}\Omega$

Solution

A useful aspect of Mathcad is that you don't have to put equations in nice "analytic" forms. You could almost write down the impedance of this circuit from inspection. We have three parallel branches, so we want to express each as an admittance (recall that Y=1/Z). First, the series R_L and L branch has admittance

$$\left(j\cdot\omega\cdot L + R_L\right)^{-1}$$

The R_C and C branches have admittance

$$R_C^{-1} \qquad \text{and} \qquad j \cdot \omega \cdot C$$

The input impedance is then the reciprocal of the sum of these admittances.

$$Z_{in}(\omega) := \left[\left(j \cdot \omega \cdot L + R_L \right)^{-1} + R_C^{-1} + j \cdot \omega \cdot C \right]^{-1}$$

Now let's look at the frequency response of this circuit.

We need to understand the dependence of the impedance and phase shift on the frequency. We know that there will be a characteristic resonance frequency. To see a wide range of values for ω, we create a range variable with equal spacing on a logarithmic scale about this frequency:

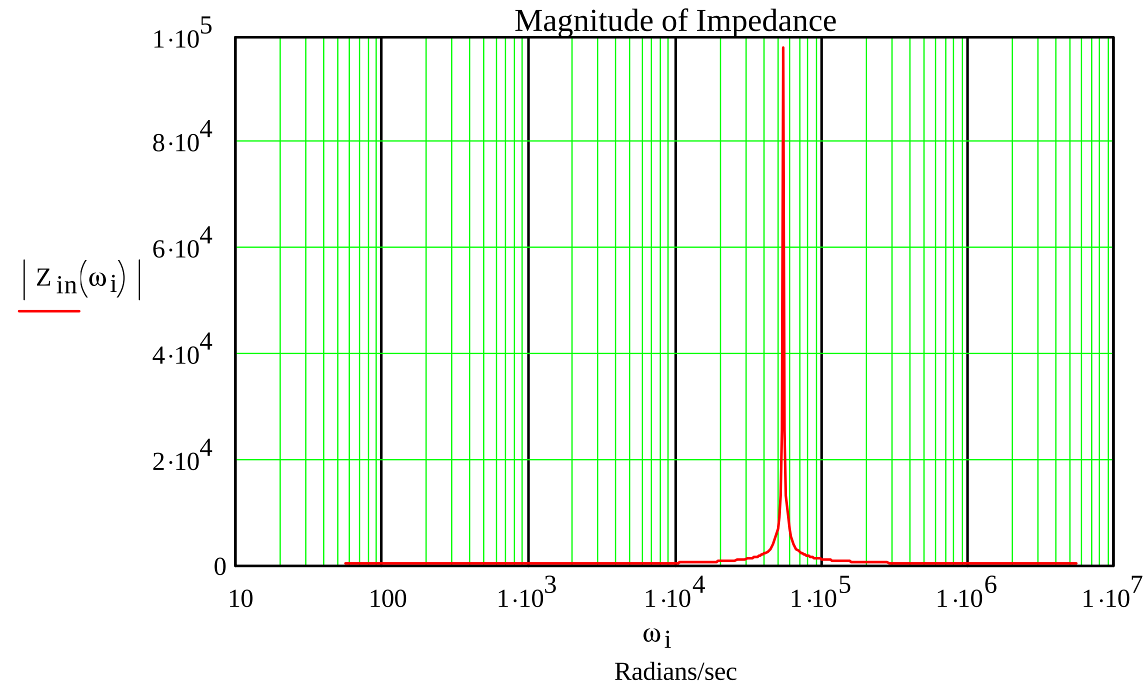
Lowest value to plot: $\omega_{low} := .001 \cdot \dfrac{1}{\sqrt{L \cdot C}}$ $\qquad \dfrac{1}{\sqrt{L \cdot C}} = 5.505 \cdot 10^4 \, s^{-1}$

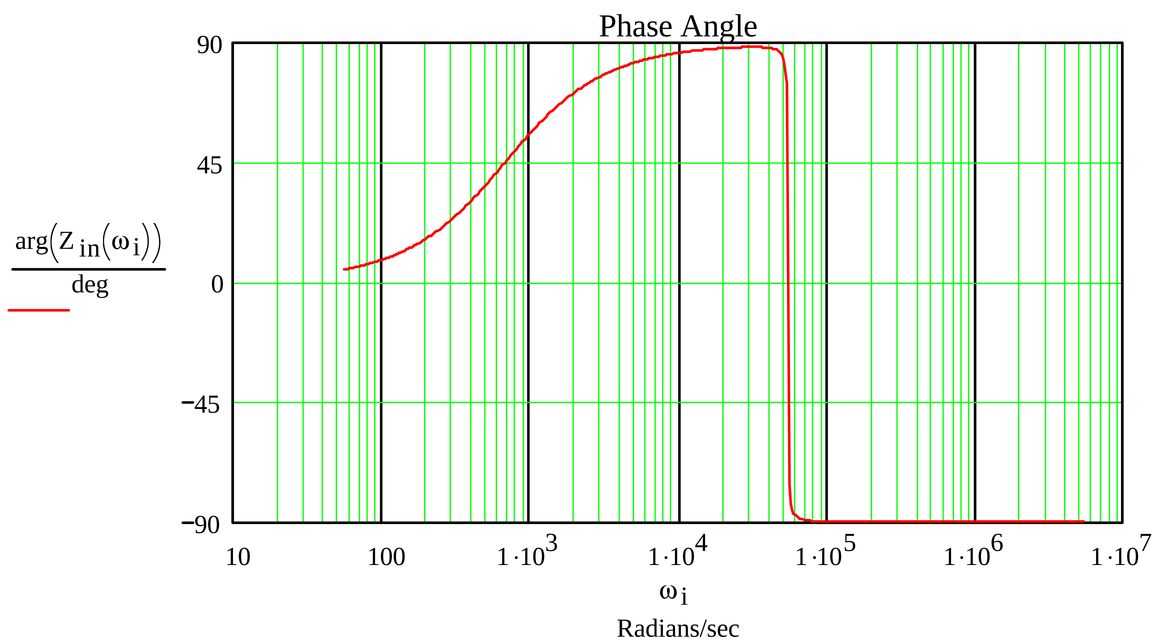
Highest value to plot: $\omega_{high} := 100 \cdot \dfrac{1}{\sqrt{L \cdot C}}$

Number of points: $N := 500$ $\qquad i := 0 .. N - 1$

Step size: $r := \log\left(\dfrac{\omega_{low}}{\omega_{high}} \right) \cdot \dfrac{1}{N}$ $\qquad r = -0.01$

Range variable: $\omega_i := \omega_{high} \cdot 10^{i \cdot r}$





This document is an excerpt from Electrical Engineering Solved Problems, an electronic book also available in the ***Mathcad Library****. Modified for PHYS 258/259 by D.S. Hamilton*

Reference:

[1] Thomas and Rosa, *The Analysis and Design and Linear Circuits, 2nd Ed.*, Prentice-Hall, Inc., Upper Saddle River, NJ, 1998.