The Fast Fourier Transform

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Suppose we have a real time-dependent signal V(t) and we wish to find which frequency components are present in V(t). We have a total of 2^m data points spanning times t=0 to t_{max}, and we have recorded data every $t_{max}/2^m$ seconds.

$m := 6 \qquad 2^m = 64$

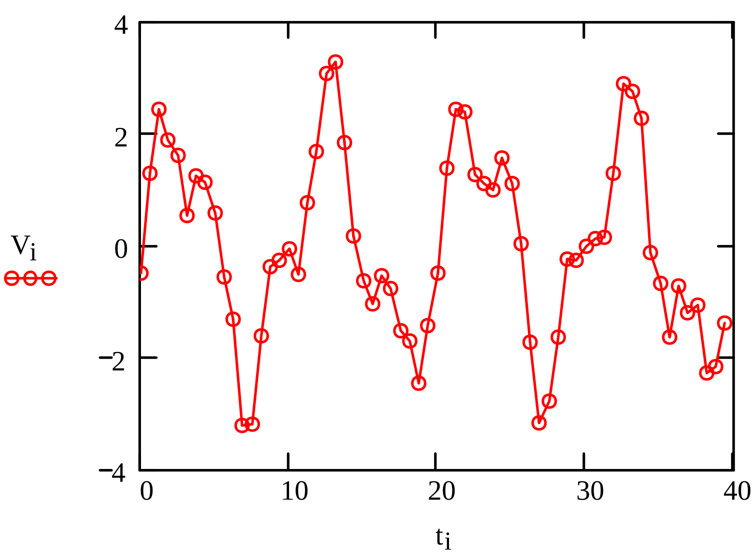
$i := 0 .. 2^m - 1 \qquad t_{max} := 40 \qquad t_i := i \cdot \frac{t_{max}}{2^m}$

$f_s := \frac{2^m}{40} \qquad f_s = 1.6$

f_s is also called the "sampling" frequency

Our "dummy" data has two frequency components, one at 0.1 Hz and the other at 0.25 Hz. We also have some random noise present in the signal. The rnd(x) function returns a uniformly distributed random number between 0 and x.

$$V_i := 2.0\sin\left(2\pi \cdot \frac{t_i}{10}\right) + 1.0\sin\left(2\pi \cdot \frac{t_i}{4}\right) + 1.0 \cdot (\text{rnd}(1) - 0.5)$$



This is what our data looks like.

To find the frequency components, take the Fast Fourier Transform of V using the built in fft function of Mathcad.

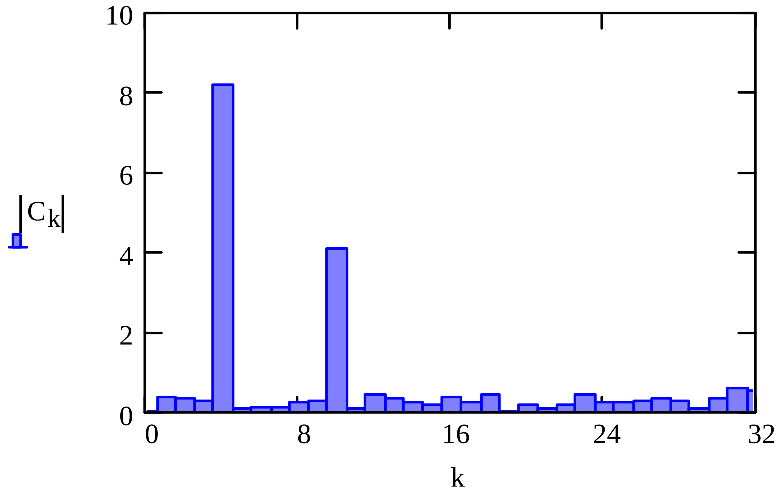
$C := \mathrm{fft}(V)$

The complex frequency vector "$C(f_k)$" is the fft of our time-dependent signal $V(t_i)$.

$N := \mathrm{last}(C)$ $N = 32$

note that C has $N+1 = 2^{m-1} +1$ elements

$k := 0 .. N$



There are two big peaks in the spectrum, one at k=4 and the other at k=10. The frequencies associated with these peaks are

$$f_4 := \frac{4}{2^m} \cdot f_s \qquad f_4 = 0.1$$

$$f_{10} := \frac{10}{2^m} \cdot f_s \qquad f_{10} = 0.25$$

Note the factor of the "sampling frequency" in the expressions above and the fact that the largest value of k is 2^{m-1}. This makes it impossible to detect frequencies larger than one-half of the sampling frequency. This is a limitation not of Mathcad, but of the underlying mathematics itself. The Nyquist frequency, which is equal to one-half of the sampling frequency, is the highest frequency that can be measured in a signal using Fourier transform techniques.