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POISSON STATITICS AND RANDOM EVENTS

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The probability distribution for counting random radioactive decay events is measured using computer-assisted data acquisition. The results are used to plot histograms of the count-rate frequency distribution, which are compared to the Poisson and Gaussian probability distribution functions.

I. INTRODUCTION

The basis for the statistical treatment of the data to be obtained in this experiment is the Poisson probability distribution, which gives the probability of observing a given number of counts per time interval. The Poisson distribution applies only if the counts occur in a random manner and the average count rate R remains constant during the measurement. The probability of measuring n counts during a time interval T is

$$P(n) = \frac{(RT)^n e^{-RT}}{n!}. \tag{1}$$

The probability function is normalized so that

$$\sum_{n=0}^{\infty} P(n) = 1. \tag{2}$$

In general, the mean value of n is defined as the first moment of $P(n)$,

$$\bar{n} = \sum_{n=0}^{\infty} n\, P(n), \tag{3}$$

and the standard deviation is the square root of the second moment about the mean,

$$\sigma = \sqrt{\sum_{n=0}^{\infty} (n - \bar{n})^2 \; P(n)}. \tag{4}$$

For the case of the Poisson distribution, the results are particularly simple:

$$\sigma^2 = \bar{n} = RT. \tag{5}$$

Note that the result for the mean number of counts is obvious, because we stipulated above that the average count rate is a constant value R.

For large values of n, the Poisson distribution of Eq. (1) is closely approximated by the Gaussian probability distribution. **Show this** by using Sterling's approximation,

$$n! \simeq n^n \, e^{-n} \, \sqrt{2\pi n} \,, \tag{6}$$

to prove that for large n,

$$P(n) \simeq G(n) = \frac{1}{\sqrt{2\pi n}} \exp\left[\frac{-\,(n-\bar{n})^2}{2\bar{n}}\right]. \tag{7}$$

(Hint: take $\ln[P(n)]$ and use the approximation $ln(1+x) \approx x - x^2/2$).

II. EXPERIMENTAL PROCEDURE

Place a radioactive source near the window of the GM tube of the Medcon counter unit. The output should be connected to the interface circuitry for the computer. Run the program ctr_evt.exe to determine the count rate. The program, which was written in the LabWindows environment from National Instruments, uses counters on the data acquisition boards both to collect the data and to control the timing. After starting the program, move the sample until the counting rate is about 10 to 20 counts per second. If you know the dead time of your detector, the time during which it is nonresponsive after each count, enter this information on the program control window—otherwise enter zero.

Now use the program to acquire data that you will use to analyze the probability distribution of the radioactive decay events. The program displays and records a "frequency-distribution" histogram, a bar graph that records the number of instances for which n events are counted during a series of fixed counting time intervals, each of duration T. You can set T, as well as the number of counting intervals to collect before the program stops, using the program control window. Experiment with the parameters for a few minutes until you both understand the behavior of the data acquisition program, and obtain a visually pleasing result.

The program allows you to store on disk both the raw counting data (the number of counts observed during each interval) and the histogram (in the form of a list of frequencies, and information on the "bins" that specify how many count values n were lumped together in each bar of the histogram). The program also displays, but does not record, the time interval, the mean count rate, and the standard deviation—be sure to record these numbers for further use. In your analysis, you will probably want to work mainly with the raw data, using Mathcad or some other spreadsheet-type program to create a new histogram suited to your specific requirements. In particular, Mathcad has functions called "hist" and "histogram" to assist you in this task. The help menu for these functions also references a Quicksheet showing an example of their use.

For your first data set, collect enough data to construct a good histogram with a value for the time window T that is roughly $2/R$, where R is the average count rate.

Now run the program a second time, but with a much higher value for the mean number of events, RT. To do this, enter a value for T that is about $15/R$. Then run the program a third time with $T = 5/R$. When you are finished, turn off the Medcom detector.

III. DATA ANALYSIS

Create a histogram[1] showing the frequency at which each count value n occurs (*i.e.*, make a histogram with a bin size of one). Normalize the area of the histogram: if $Y(n)$ is the height of the n^{th} bar, meaning that n counts were observed during an interval of duration T on Y different occasions, you should multiply all of your results by a constant C, chosen so that

$$C \sum_{n=0}^{\infty} Y(n) = 1. \tag{8}$$

In practice, the sum extends only to a finite value of n, because there will be some value n_{max} that is the largest number of counts that you ever observed during any time interval T, so for all higher values of n, $Y = 0$.

The normalized histogram can be interpreted directly as an estimate of the normalized probability distribution $P(n)$ for observing n events during time T. Include error bars with each data point (you may want to discuss with your TA how to go about estimating these). Calculate $\bar{n}$, the mean number of counts per time interval, and its uncertainty, from the total accumulated count, the total elapsed time, and the value of the time interval. Then calculate a value for $\bar{n}$ directly from Eq. (3).

Compare each set of data to the Poisson distribution function. For the large n data set(s), also compare your results to the Gaussian distribution. To facilitate this comparison, overlay graphs of these distribution functions on your data sets. Calculate the standard deviation σ from your value of $\bar{n}$ as well as from Eq. (4). Indicate $\bar{n}$ and σ on your graphs.

Also calculate the mean count rate R in counts/sec from your results.

How well do the two distribution functions describe the data? Do the deviations between the Poisson function and your results correspond well with the size of your error bars?

[1] The data acquisition program records the number of times that n events are counted in a fixed time window of duration T. An array $Y(n)$ is used to keep track of how many times n counts were recorded. For example, if 12 decays are counted during the time interval of duration T, then the array element $Y(12)$ is incremented by 1, and if 7 decays are counted then $Y(7)$ is incremented. The probability $P(n)$ for recording n counts in the fixed time interval is then simply proportional to the array $Y(n)$.