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THE CURRENT BALANCE
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The time average force between two parallel conductors carrying an alternating current is measured by balancing this force against the gravitational force on a set of known masses. The relationship between the time-average force and the root-mean-square current is investigated using several methods of analysis.

I. INTRODUCTION

The MKS system of units defines the ampere in terms of the force between two parallel conductors both carrying a current: "One ampere is that unvarying current which, if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience of force of exactly 2×10^{-7} newton per meter of length."

The current to be measured in this experiment is passed in opposite directions through two parallel horizontal bars that are connected in series. The lower bar is fixed and the upper bar is balanced a few millimeters above it on a pair of knife-edges. The upper bar has a small pan into which analytical weights are placed, thereby causing the upper bar to drop down toward the lower one. When the current is turned on and increased sufficiently, the repulsive force between the two bars causes the upper bar to rise back up to its equilibrium position. At this point, the repulsive force is equal and opposite to the gravitational force on the analytical mass. The position of the bar is observed by means of a mirror mounted on the balance beam and a He-Ne laser and meter stick placed a few meters from the mirror.

II. THEORY

Consider two parallel straight rods, each of length L and separated by a distance d as shown in Fig. 1. The current I along the z axis in the lower rod produces a magnetic field $\vec{B}$ at the upper rod of

$$\vec{B} = -\frac{kI}{d}\,\hat{x}, \tag{1}$$

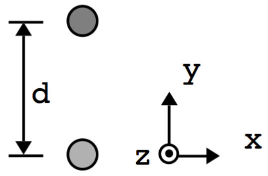

FIG. 1: The current in the lower rod is in the $+\hat{z}$ direction, whereas it is along $-\hat{z}$ in the upper rod.

where $k = \mu_o/2\pi = 2 \times 10^{-7}\,\mathrm{N/A^2}$. This field creates a force on the upper rod that is carrying a current I in the $-\hat{z}$ direction. This force is

$$\vec{F} = IL(-\hat{z} \times \vec{B}) = \frac{kIL}{d}\,\hat{y}. \tag{2}$$

For a sinusoidal time dependence of the currents, i.e. $I(t) = I_0 \sin(\omega t)$, the time average force is

$$\langle F \rangle = \frac{kL}{d}\left[\frac{1}{\tau}\int_0^\tau I_0^2 \sin^2\left(\frac{2\pi t}{\tau}\right) dt\right], \tag{3}$$

where we have integrated over one period $\tau = 2\pi/\omega$. The term in the brackets is I_{rms}^2 and thus Eq. (3) can be written as

$$\frac{\langle F \rangle}{I_{rms}^2} = \frac{kL}{d}. \tag{4}$$

The total magnetic field at the upper rod is comprised of the harmonic field as described above plus any static stray fields, such as the earth's magnetic field. Expand the analysis to include a static field B_s and **show that** Eq. (4) is also valid in this case. This is why we use an AC current instead of a DC current for this experiment.

III. PROCEDURE

The current balance is a delicate instrument and should be handled carefully. Particularly sensitive to damage are the two knife edge suspension points and the surfaces upon which they rest. The upper balance assembly can be raised off of the knife edges by a lift mechanism. The electrical wiring schematic is diagramed below. The transformers $\mathtt{T}_1$ and $\mathtt{T}_2$ have an adjustable output from 0 to 135 volts AC for a 115-volt AC input and $\mathtt{T}_3$ is a fixed step-down transformer with a 6.3/115 ratio. The ammeter is a calibrated for a root-mean-square reading. Also note that it has a reflective strip that is used to prevent parallax errors in

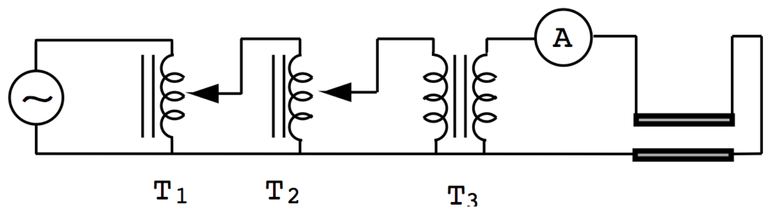

FIG. 2: The wiring diagram for the current balance.

reading the needle position. Before you take any measurements, be sure to level the base by using the two thumb screws at front of the unit.

Use the lift arms to reposition the balance assembly on the posts. Practice this a few times so that you can do this smoothly. With no mass in the pan and no current flowing, the gap between the two rods should be a few millimeters. If it is not, carefully adjust the counter-weight behind the mirror. Then use the lift arms to reposition the balance assembly. Be careful to never touch the unit or make any mechanical adjustments to the apparatus while the current is flowing.

Place the 500 mg mass in the pan, which should bring the rods into contact. Record the position on the He-Ne laser beam on the meter stick. Carefully remove the mass and record the new position of the laser beam when the balance assembly is in equilibrium. Practice placing and removing the mass until you can do it smoothly, without jarring the balance. If you do bump it, use the lift arms to reposition the knife-edges. Also note that the balance is susceptible to small air currents.

With the power to the transformers turned off, set T_2 to its minimum value and T_1 to about full scale. Turn the power on and slowly increase T_2 until the ammeter reaches 20 A. (If the fluctuations in the current reading are more than 0.1 A out of 20 A, notify your instructor. These fluctuations can be caused by Ohmic heating coupled to a temperature dependent electrical resistance.) You should observe the result of the repulsive force between the parallel rods as indicated by the movement of the He-Ne laser spot. Use T_1 to adjust the current (never exceed 20 A) leaving T_2 fixed.

Set the current to zero and carefully place the 50 mg mass in the pan. Slowly increase the current until the laser beam points to the equilibrium spot. Record the value of the current. Remove the mass and turn the current down to zero. Verify that the laser spot returns to the same equilibrium position. If not, use the lift assembly to reposition the knife-edges and record the new positions for when the rods are touching and when the balance is in equilibrium and repeat this step.

You should measure the current required to bring the balance to its equilibrium position for masses of 50, 100, 150, 200 and 250 mg. Verify that the zero-current and zero-mass equilibrium position of the laser spot has not shifted during the placement or removal of the masses. It is good experimental technique to calculate mg/I_{rms}^2 (which should stay nearly constant) as you go along. When you have a good set of data points, adjust the counter-weight slightly to give you a different equilibrium position and then take a second set of data.

Record the following distances: (i) L, the length of the upper rod as measured from center to center of its two supporting rods. (ii) a, the lever arm, from the center of the front bar to the knife edge. (Do this for both sides and use the average.) (iii) b, the distance from the mirror to the meter stick scale. (Note that the mirror is silvered on its back surface.) (iv) d_u, the diameter of the upper rod. (v) d_l, the diameter of the lower rod.

IV. DATA ANALYSIS

From your data calculate the mean value of $\langle F \rangle / I_{rms}^2$ and its uncertainty. Plot $\langle F \rangle$ as a function of I_{rms}^2. Draw a straight line on your graph that passes through the origin and has a slope m equal to the mean value calculated above. Use dashed lines to draw additional lines whose slopes are $m + \Delta m$ and $m - \Delta m$.

A simple geometric argument shows that the distance d can be calculated from

$$d \approx \frac{Da}{2b} + \frac{d_u + d_l}{2}, \tag{5}$$

where D is the difference between the *just touching* and the *equilibrium* readings for the position of the He-Ne laser beam spot on the meter stick. Use this value of d and the measured value of L to calculate the right hand side of Eq. (4). Compare with your mean value of $\langle F \rangle / I_{rms}^2$ from above.

Another way to analyze your measurements of $\langle F \rangle$ as a function of I_{rms}^2 is to do a *least squares fit* of the data to a straight line of the form $y = a + bx$. The value of a that you calculate should be small, but may not be exactly zero. Since Eq.(4) predicts a linear relationship between $\langle F \rangle$ and I_{rms}^2 with an intercept of zero, you should try a least squares fit to an equation of the form $y = cx$. First you must minimize the variance,

$$\sigma^2 = \frac{1}{N}\sum_i (y_i - cx_i)^2 \tag{6}$$

with respect to c and **show that**[1] the best fit value of c is

$$c = \frac{\sum x_i y_i}{\sum x_i^2}, \tag{7}$$

and that the uncertainty in c is

$$\sigma_c = \sigma\sqrt{\frac{1}{\sum x_i^2}}. \tag{8}$$

Use these results to fit $\langle F \rangle$ against I_{rms}^2. Compare to your previous values from above.

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[1] These results are excercise problems 8.5 and 8.18 in: John R. Taylor, *An Introduction to Error Analysis*, 2nd Ed. (University Science Books, 1997).