Operational amplifiers

Assume ideal for now

Usual Version

comp (usually optional or absent)

$+V$ (usually 15)

actual connections:

$-V$ (usually, -15)

bal. (optional, sometimes absent)

(no ground)

Properties (we will look at real op amps later)

1) No input current

$\text{or } \Delta V_{in}$

2) Output = $+ - (-) \times \infty$  ($\text{in } 0$)

Intended for use with feedback:

Follower illustrates

Output will always try to set itself

So $++$ inputs are at same $V$. To see, assume $+$ at $1V$, $-$ at $1.1V$
(In fact, this works even with finite gain, so long as \( Z_{in} = \infty \).)

Compliance range:
Typically from \(-V+2 \) to \(+V-2 \) for older designs.
"Special" types go all the way to one or both supplies within \( \pm 10 \text{ mV} \) or so.
Now quite common due to battery-powered devices.

Basic amplifiers (DC coupled):

**Non-inverting:**

\[
\begin{align*}
V_+ &= V_{in} \quad \text{(due to (1) feedback)} \\
V_- &= \frac{R_1}{R_1 + R_2} V_{out} \quad \text{(exactly, since)} \quad I_{op amp} = 0 \\
\text{So} \quad V_{out} &= \frac{R_1 + R_2}{R_1} V_{in} \\
\frac{V_{out}}{V_{in}} &= 1 + \frac{R_2}{R_1}
\end{align*}
\]

\( Z_{in} = \infty \) (for LF 411 ~ 10\(^{-3}\) \Omega)
\( Z_{out} \) very low, at least for low freq., small signals.

**Inverting**

\[
\begin{align*}
I_1 &= -I_2 \\
V_{in}/R_1 &= -V_{out}/R_2
\end{align*}
\]
So gain
\[
\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_2}{R_1}
\]

\[
\begin{align*}
Z_{\text{in}} &= R_1 \\
Z_{\text{out}} &= 0
\end{align*}
\]

Most common configuration. This is partially because of the "virtual ground" at (O).
It lets us build a

**Summing amplifier**

\[
\begin{align*}
\text{(-) Still at ground, so } I_{\text{for}} &= 0 = I_a + I_b + I_c - I_2 \\
\text{and } V_{\text{out}} &= -R_2 \left( \frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} \right)
\end{align*}
\]

**Variant:**

\[
\begin{align*}
(V_1 - V_2)V_{\text{in}} &= -(V_{\text{out}} - V_2)V_{\text{in}} \\
\frac{V_{\text{out}}}{R_b} &= \frac{V_2}{R_b} + \frac{V_2 - V_c}{R_a} \\
V_{\text{out}} &= (V_2 - V_{\text{out}}) \frac{R_b}{R_a} + V_2
\end{align*}
\]
The circuit is a differential amplifier, but a bad one.
However, inputs are asymmetric, and output is not to ground, but rather to \( V_2 \).

Better differential amplifier ---

\[
\begin{align*}
V_1 - V_2 \frac{R_2}{R_1 + R_2} &= -\frac{V_{\text{out}} - V_2}{R_2} \\
V_{\text{out}} &= \frac{V_2}{R_1 + R_2} + \frac{V_2}{R_1 + R_2} - \frac{V_1}{R_1} \\
&= \frac{V_2 R_1 + V_2 R_2 - V_1 (R_1 + R_2)}{R_1 (R_1 + R_2)} \\
&= \frac{(V_2 - V_1) (R_1 + R_2)}{R_1 (R_1 + R_2)} \\
\frac{V_{\text{out}}}{R_1} &= \frac{R_2}{R_1} (V_2 - V_1)
\end{align*}
\]

But, need precisely matched resistors!

Problem is common-mode rejection ---

\[
\begin{align*}
V_0 + E &\quad \text{if not matched} \\
V_0 &\quad \text{Can be addressed by hand matching, laser trimming, or instrumentation amplifiers.}
\end{align*}
\]
Linearized power amplifier ---

\[ \text{(In practice, add protection, bias, \text{ and} \text{ thermal \ stabilization).}} \]

Can do same with HV output:

\[ \frac{V_{out}}{1000} = 10 \times V_{in} \]

\[ \sim V_{out}/1000 \quad \text{Note F.B. \gg 1k needed.} \]

Even better; use single-chip solutions such as:

1) Texas Instruments OPA548T: up to 60V, up to 3A, $\sim 15$

2) Apex/Cirrus PA740CC: up to 350V, up to 0.12A, $\sim 25$

Transimpedance amplifier:

\[ V_{out} = -R \times i_{in} \]

\[ Z_{in} = ? \quad (0) \]
This is used with photodiodes and similar detectors.

\[ I \propto \text{photon flux} \]

(\text{efficiency about } 50\%)

At 633 nm:

\[ E = h \nu = h \frac{c}{\lambda} \]

\[ = (6.63 \times 10^{-34} \text{ J s}) \frac{2 \times 10^{-14} \text{ J}}{633 \times 10^{-9} \text{ m}} \]

\[ = 3.14 \times 10^{-19} \text{ J} \]

0.1 mW \( \Rightarrow \) \( 7 \times 10^{-14} \text{ V s} \) or \( 3.2 \times 10^{14} \text{ V/s} \)

Curves of \( P = 5 \times \left( \frac{3.2 \times 10^{-14}}{1.6 \times 10^{-14}} \right) \left( 1.6 \times 10^{-14} \frac{\text{V}}{\text{A}} \right) \)

\[ = 26 \mu \text{A} \text{ or } 0.26 \frac{\text{A}}{\text{W}} \]

Sometimes used instead in \textbf{photovoltaic mode},

\[ V = \text{I}_P R \] (upto \( \frac{1}{3} \text{ max} \) or less)

Often used with reverse bias, to reduce capacitance:

\[ \frac{V_{\text{out}}}{(\text{to } -14 \text{V})} \]

or \( V = \frac{1}{2} \int_{-0.15}^{0.15} \)

The op amp helps to minimize effects of stray \( C_s \) too.

Since \( Z_i \approx 0 \), the time constant \( RC \) does not apply.

Instead, a detailed analysis shows that

\[ f_{\text{3dB}} \approx \sqrt{\frac{f_t}{2 \pi C_{\text{dine}} R_f}} \]

\( f_t = \text{unity-gain frequency} \) or "gain-bandwidth product".

For a perfect op amp, there would be no limit!
Arbitrary feedback:

\[ f(V_{out}) = V_{in} \]

So \( V_{out} = f^{-1}(V_{in}) \)

Can also do with \((-)\) in, if device in loop produces a current rather than a voltage output:

\[ V_{in} = \frac{V_{out}}{R_{in}} \]

\[ \frac{V_{in}}{R} = -f(V_{out}) \]

\[ V_{out} = f^{-1}\left(\frac{V_{in}}{R}\right) \]

So if \( g = e^V \), \( V_{out} = -\ln\frac{V_{in}}{R} \) (see H&H for such a circuit)

Ex: integrator

\[ \frac{V_{in}}{R} = -C \frac{dV_{out}}{dt} \]

\[ V_{out} = \left[-\frac{1}{RC}\int_{0}^{t} V_{in} \, dt + V_{initial}\right] \]

This is exact! The virtual ground keeps it that way.

(But in real op amps, also get)

\[ \int V_{offset} \]
Integrator needs a reset, or a resistor to set a finite ac gain, unless in a closed loop: $R_2$ (large)

High freq: $\frac{1}{RC} \int \text{Vin} \, dt$
Low freq: $-\frac{R_2}{R_1}$

(gives a gain, in effect) often summed w/ linear gain.

Differentiator: same basic theme, but C not in feedback:

Ideal: $\frac{C}{R} \frac{dv_{in}}{dt} = -\frac{V_{out}}{R}$

$V_{out} = -RC \frac{dv_{in}}{dt}$

Actual (stable): $C = 50 \mu F$, $R = 100 \Omega$:

Integrator for large W
Filters

Instead, multi-pole active filters are used; can synthesize any RLC filter without loading poles!

Basic Sallen & Key Filter

"Bootstrapping" sharpens rise

Oscillators — classic cheap example is

Simples is relaxation osc —

\[ T \approx 2.2RC \]
Real Op Amps I

AC Op amp behavior

Finite gain instability.

Recall the behavior of an RC low-pass:

\[ \begin{align*}
\log V_{\text{out}} & \quad \downarrow \\
\frac{1}{\text{ac}} & \quad \downarrow \\
\log W & \quad \downarrow \\
\phi & \quad \uparrow \\
0 & \quad 0 \\
-90^\circ & \quad \phi
\end{align*} \]

\[ \text{db/octave} \]

Most op-amps have a dominant "pole" so they look very similar...

\[ \begin{align*}
\log \text{gain} & \quad \downarrow \\
100 \times & \quad 0 \\
\log W & \quad \downarrow \\
\phi & \quad \uparrow \\
0 & \quad 0 \\
-90^\circ & \quad \phi
\end{align*} \]

"Unit gain" frequency

\[ x \times (-1) \text{ if inverting} \]

- Causing loss of accuracy

But what if we have additional phase lag?

If \( |\Phi| > 180^\circ \), neg. feedback \( \rightarrow \) positive, so we get instability, and excursions at \( W \) will be amplified \( \rightarrow \) oscillation!
Look at these problems in turn:

1) Finite gain. As an example, look at a non-inverting voltage amp (other configs. are similar):

\[ V = B \times V_{\text{out}} \]

\[ V_{\text{out}} = A \left( V_+ - V_- \right) = A \left( V_{\text{in}} - B V_{\text{out}} \right) \]

\[ V_{\text{out}} = \frac{A}{1 + AB} V_{\text{in}} \tag{1} \]

If \( A = \infty \), \( G = \frac{1}{B} \) \( \left( V_{\text{out}} = \frac{1}{B} V_{\text{in}} \right) \)

But now, \( V_{\text{out}} = A \left( V_+ - V_- \right) = A \left( V_{\text{in}} - B V_{\text{out}} \right) \)

\[ V_{\text{out}} = \frac{A}{1 + AB} V_{\text{in}} \tag{1} \]

So, if we have a "single-pole" frequency rolloff, 6 dB/octave,

\[ \log G \]

\[ \text{open-loop, } A \]

\[ \text{closed-loop, } \frac{A}{1 + AB} \]

\[ \log f \]

\[ \text{open} = \text{closed} \approx AB \]

So, (a) Finite gain changes \( G \)
(b) Frequency response is limited by gain rolloff.

Also, both \( Z_{\text{in}} \) and \( Z_{\text{out}} \) are diminished affected

If we have for open-loop, \( \frac{1}{R_{\text{in}}} \)

Then for closed loop,

\[ I_{\text{in}} = \frac{V_{\text{in}} - B V_{\text{out}}}{R_{\text{in}}} = \frac{V_{\text{in}}}{R_{\text{in}}} \left( 1 - \frac{AB}{1+AB} \right) = \frac{V_{\text{in}}}{R_{\text{in}} (1+AB)} \]

Thus \( Z_{\text{in}} = \frac{V_{\text{in}}}{I_{\text{in}}} = (1+AB) R_{\text{i}} \) (not \( \infty \))

Likewise, can show \( Z_{\text{out}} = \frac{1}{1+AB} R_{\text{o}} \) (not zero) open-loop output imp.
2) Stability (a major issue in real designs)

If there happen to be two poles, due to external components,

\[ \log G \sim 6 \text{ dB/octave} \]

\[ \log f \sim 12 \text{ dB/octave} \]

see below

\[ -180^\circ \]

Oscillation if \( \text{gain} > 0 \text{ dB} \)

with \( |G| > 180^\circ \)

Assuming lead/feedback are well-behaved, worst case is unity gain, because what counts here is the loop gain \( AB \), which determines how much is fed back. (It's also \( \frac{\text{open-loop gain}}{\text{closed-loop gain}} \).)

Compensation is used to assure stability.

Basic idea: make sure that closed-loop gain for \( A = \infty \) intersects open-loop gain with slope \( < 12 \text{ dB/octave} \).

Differentiator obviously horrible, needs lots of work to assure stability.

The 411, like the 741, is already unity-gain compensated internally.

Note: poles, zeroes refer to response function & Laplace transform language.

ac coupling: use when appropriate to eliminate offsets

\[ \text{or} \]

\[ \text{offset} \]
1) Input offset voltage: \( V_o \) held at \( V_+ - V_{os} \), not \( V_+ \). In a follower, replicated at output:

\[
\text{Vin} \quad V_{out} = V_{in} + V_{os}
\]

In amplifiers, can be amplified

\[
1k \quad V_{out} = 100 \times V_{os}
\]

LF411: 0.8 mV typ., \( \pm 7 \mu V/\degree C \) has trimming
741: 2 mV \( \pm 10 \mu V/\degree C \) \( \leq \) provisions.
AD8551: 0.001 mV \( \leq 0.005 \mu V/\degree C \)

2) Offset & bias current

\[
|I_-| = |I_+| = \text{bias current}
\]

Ignoring \( V_{os} \), \( V = AV = I_+ (R_1 / R_2) \) rel. to perfect op amp.

\[
\Delta V_{out} = -\frac{R_2}{R_1} \Delta V
\]

\( \approx \frac{R_2}{R_1} I_+ \) if \( R_2 \gg R_1 \)

741: 500 nA
LF411: 0.2 nA
AD8551: 60 pA (1) \( \not\leq \) pA!

Can often be compensated:

\[
\frac{R_2}{R_1} \quad \frac{R_1}{R_2}
\]

\[
\text{741: 200 nA}
\]
LF411: 0.1 nA

However, there's still the input offset current \( I_{os} = I_+ - I_- \).

For many modern op amps, not that much smaller than \( I_{bias} \).

For ac signals, ac coupling eliminates.
3) Slew rate = response to large-signal step function

\[ \text{Determined by current limitations of output stages.} \]
\[ \neq \text{bandwidth} \]

- 741: 0.5 V/\mu s
- LF411: 15 V/\mu s
- LF311: \approx 25 V/\mu s (with 500 \Omega yielding to 15)
- AD8009: 5500 V/\mu s

4) Output swing: "rail to rail" of amps can drive to \(< 20 \text{ mV wrt supply voltages} -
\text{can run off a single 1.5 V battery.}\)

5) Common mode rejection -- typ. \(>100 \text{ dB}\), usually limited by external circuitry.

6) Finite gain and phase shifts \( \Phi(\omega) \) -- to be discussed shortly.

7) Noise:
   1. low-frequency
   2. white noise \((nV/\sqrt{Hz})\)

\[ \text{Will discuss later.} \]
"Superdene"

to draw some current if no load is connected or sink current if necessary.

Vin

Note: Zout = 10 k if not conducting.

Full-wave variations (see H&H) can be used for measuring V', etc., or for calculating |Vin|.

Improved:

(0) in ⇒ D1 conducts, gain = -1

(†) in ⇒ D1 reverse-biased

D2 clamps V' at -6V so V'' = 0.

Here Vout = 0V, but impedance ~ R. ⇒ use follower if needed.
Comparators

They are really just op amp-like devices meant to operate without feedback, or even with (+) feedback. Typically, they switch very quickly and cleanly between two output states.

\[ V_{i} \rightarrow V_{o} \]

\[ \text{when } V_{i} < V_{t} \]

Fast slow rate, flexible output stage

\[ \text{when } V_{i} > V_{t} \]

LM311 is a classic comparator (open collector output needs pullup)

A Schmitt trigger is often used to detect logic transitions on a noisy signal line:

Often built into line receivers. Can be manually set using a comparator and resistors — for example,

Assume \( R_{\text{pullup}} << R_{3} \) (Power on \( V^{+} = \text{pin8}, V^{-} = \text{pin 4} \))

Parallels \( R_{2} \) if output at 0V, so

\[ V_{\text{ref}} = V_{\text{LO}} = \frac{R_{2}}{R_{1} + R_{2}} (15 \text{V}) \frac{R_{2}}{R_{1} + R_{2}} \]

Parallels \( R_{1} \) if output at 15V, so

\[ V_{\text{HI}} = \frac{R_{2}}{R_{1} + R_{2}} (15 \text{V}) \]

and if \( R_{3} \) is fairly large, \( V_{\text{HI}} > V_{\text{LO}} \)
Typ. control system:

- Sometimes a differentiator is added - "PID" controller
- Why are buffers used?
- Why is integrator there?
- Probably not stable as shown!
- In practice, usually use a thermistor + voltage source for $T_0$, and an instrumentation amp.