I. Kepler's Laws and their proof from principals of Newtonian physics

A. **The square of the period of a planet in a near circular orbit is proportional to the cube of its distance from the sun**, or $T^2 = A R^3$. Proof: the centripetal force making a planet move in a circular orbit around the sun is the gravitational force. Equate the centripetal force to the gravitational force, substitute $v = r \omega$ for the magnitude of linear velocity tangent to the circle in the term for centripetal force and remember that the angular velocity $\omega = 2 \pi / T$.

B. **The motion of any body orbiting the sun sweeps out equal areas in equal time intervals of its orbit.** Proof: Sketch an ellipse and show that areas of triangles dA in time intervals dt are such that dA/dt = a constant related to the angular momentum, which is conserved

C. **Orbits of planets about the sun are ellipses.** Proof: using conservation of energy write K.E. + P.E. = E, where E is constant. Substitute K.E. and gravitational P.E. in terms of masses and velocities, convert velocity to rates of change of a position vector r in polar coordinates, include a constant for angular momentum, change variables of differentiation, and show that the energy conservation equation can be converted into a differential equation for the rate of change of radius w.r.t. angle in polar coordinates. Integrate this equation and show that it is the equation of an ellipse in polar coordinates. Eccentricity of the ellipse must be less than 1 for a bound orbit, requiring the magnitude of P.E. to be greater than the magnitude of K.E. Otherwise the polar equation describes a parabolic orbit for eccentricity = 1 and a hyperbolic orbit for eccentricity greater than 1. Click here for details.

II. The center of mass system and its importance.

A. An accurate periodic law must be derived in a center of mass system.
B. All objects in an N body system orbit about a common center of mass.
C. Other solar systems may be detected from oscillations of a star toward and away from an Earth observer caused by the star orbiting the center of mass of its solar system. These oscillations may be measured from Doppler shifts of its spectrum. Click here for an example.

III. N body problem stable/unstable points

A. Stable points of a 3 body celestial mechanic problem are called Lagrangian points. Objects tend to be stable in these points. Example: Trojan asteroids near Jupiter. Click here for Earth/Moon region.

B. Unstable points are points in a 3 body problem tend to move orbiting material away from those points. Example: Kirkwood gaps in the asteroid belt between Mars and Jupiter. Click here example plot.

C. Stable and unstable points of a 3 body may be found be calculating rates of change of positions of a 3rd body in a center of mass coordinate system.