

Measuring properties of superposed light beams carrying different frequencies

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Abstract: When two electromagnetic fields of different frequencies are physically superposed, the linear superposition equation implies that the fields readjust themselves into a new mean frequency whose common amplitude undulates at half their difference frequency. Neither of these mathematical frequencies are measurable quantities. We present a set of experiments underscoring that optical fields do not interfere with each other or modify themselves into a new frequency even when they are physically superposed. The multi-frequency interference effects are manifest only in materials with broad absorption bands as their constituent dipoles attempt to respond collectively and simultaneously to all the optical frequencies of the superposed fields. Interference is causal and real since the dipoles carry out the operation of summation dictated by their quantum mechanical properties.

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OCIS codes: (260.3160) Interference; (999.9999) Interference, multi-frequency; (140.4050) mode locked laser.

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1. Introduction

Dirac states in his book [1] that a photon interferes only with itself, interference between different photons never occurs. The purpose of this experimental paper is to clarify the fundamental concepts behind the observable effects when electromagnetic fields of different coherent frequencies are literally superposed, with a self-consistent, semi-classical argument that does not require the concept of a photon.

We define the frequency of the EM field as that frequency at which the electric vector oscillates and what Mandel [2] defines as the instantaneous frequency and some others call carrier frequency. Both the traditional multi-frequency laser mode locking [3] and two-frequency beat [4] phenomena fall under the category of superposition of light beams carrying different optical frequencies. Our key objective is to give qualified support to the established view that light beams of different frequencies do not interfere by themselves. However, they do impose the interference effects on the excitability of dipoles in the materials used as sensing tools. EM fields, by themselves, do not produce the mode-locked pulses or beat signals; the materials do by virtue of their excitation and absorption characteristics in response to the presence of multi-frequency fields. However, while the beat signal does not exist independently (outside the detector), the mode locked pulses, once generated, are real, propagating time pulses of the EM field. Beat signals are now used routinely as a precision tool for measuring surface quality, distance, frequency of light beams, stabilizing a laser frequency, etc. [5-11]. Mode locking, of course, has become the most important tool for controlling laser energy that facilitates applications from delicate, living gene manipulation to ablating hardest materials on the Earth.

Let us start with the generalized equation representing a superposition of N periodically separated optical frequencies, without any arbitrary phases. This represents a simple mode locked laser [3],

$$E(t) = \sum_{n=1}^N e^{2\pi i(\nu+n\delta\nu)t} = e^{2\pi i\nu t} \frac{\sin(N+1)\pi\delta\nu t}{\sin\pi\delta\nu t}, \quad (1)$$

where ν is the mean of the N periodic frequencies and $\delta\nu$ is the periodic laser mode spacing. Let us now consider the conceptually simplest case of superposition of two different coherent frequencies [12, 13], or the conditions for locking two modes ($N = 2$):

$$E(t) = \cos(2\pi\nu_1 t) + \cos(2\pi\nu_2 t) = 2\cos(2\pi\nu t)\cos(2\pi\nu_m t), \quad (2)$$

where, the mean frequency, $\nu = (\nu_1 + \nu_2)/2$ and half the difference frequency, $\nu_m = (\nu_1 - \nu_2)/2$. The basic question is whether the EM fields of different frequencies, when physically superposed, carries out the "summation" operation of the left-hand-side of Eqs. (1) and (2) and transform themselves into the right-hand-side as a new E-vector oscillating at the mean frequency of ν whose amplitude modulates at a frequency, $\delta\nu/2$ as a "grating" function for Eq. 1, and at ν_m as a cosine function for Eq. (2). From the viewpoint of an experimentalist, the question can be rephrased as whether ν and $\delta\nu/2$ (or, ν and ν_m) are real and measurable. Accordingly, we have carried out the following four experiments to measure the properties of superposed light beams carrying two coherent, but different frequencies represented by Eq. (2).

First, let us explain the overall experimental set up shown in Fig. 1. We have chosen a tunable, external cavity, very narrow line, CW 780 nm laser that can be made to resonate to any one of the Rb-resonant fluorescence lines. This laser line is sent through a 2GHz acousto-optic modulator (AOM) to introduce 2 GHz Doppler shift in the optical frequency. The AOM gives back the original, undeviated laser beam ν_L and a diffracted, Doppler shifted beam ν_S . The beams ν_L , ν_S , and $(\nu_L+\nu_S)$ are then separately sent through three separate Rb-vapor tubes (shaded yellow box). This superposed beam $(\nu_L+\nu_S)$ can also be thought of as the simplest but synthetic “mode locking” in a two-mode laser.

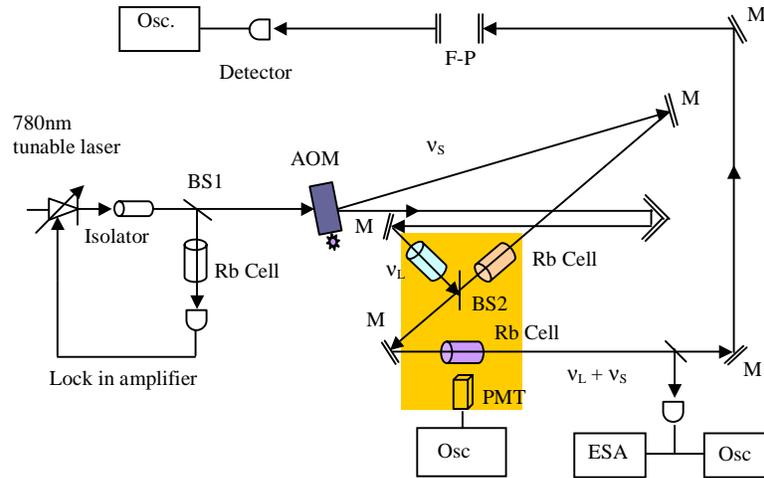


Fig. 1. The schematic diagram shows a set up for four different experiments. A tunable 780nm source at the left generates ν_L . The AOM generates a Doppler shifted line ν_S . The three Rb cells, in a shaded yellow box, separately receive ν_L , ν_S , and superposed (ν_L, ν_S) beams. The superposed beam is then further analyzed by optical Fabry-Perot (F-P) and electronic spectrum analyzers (ESA) using high speed detector and monitored by oscilloscopes (Osc.).

2. Measuring the properties of the superposed beams

2.1 Experiment 1. A high-speed detector observes beat signal $(\nu_L-\nu_S)$.

The superposed beams were directed onto a high-speed detector and we registered the beat signal $(\nu_L-\nu_S)$, (i) directly by a high-speed, photoconductive detector and an oscilloscope (HP Model 86100A) and also (ii) by the same detector and an electronic spectrum analyzer. The beat signal vanished when any one of the two beams was blocked before the beam combining beam splitter. This straightforward result is well understood since the time of Forrester [4]. But for extra clarity, let us recall the basics. Equation (2) does not represent anything directly observable or measurable. Please, note that the detected beat frequency is $(\nu_L-\nu_S)$, and not $(\nu_L-\nu_S)/2$, as indicated by Eq. 2. The measurable quantity is the energy absorbed by the dipoles in the detector during interaction with the superposed fields. It is proportional to the time (or ergodic ensemble) average of the square of the electric field amplitude or, $\langle |E(t)|^2 \rangle$ [12, 13]:

$$\langle |E(t)|^2 \rangle = \langle |\cos(2\pi\nu_L t) + \cos(2\pi\nu_S t)|^2 \rangle \approx 1 + \cos[2\pi(\nu_L - \nu_S)t] \quad (3)$$

It is instructive to note the significance and convenience of complex representation. The RHS of Eq. (3). could have been arrived at by simple square modulus, had we written Eq. (2) in complex representation, as in Eq. (1). For a high-speed photo-conductor, $\langle |E(t)|^2 \rangle$ is the probability of the rate of excitation of electrons from the valence to the conduction band,

allowing the electric current to flow under an appropriately applied potential difference (see Fig. 2(a)). So, the photo detector detects the beat frequency $(\nu_L - \nu_S)$ as in Eq. (3) and not $(\nu_L + \nu_S)/2$ as implied by ν_m in Eq. (2). For a saturable absorber, $\langle |E(t)|^2 \rangle$ represents the probability rate of electronic excitation from lower to the upper band that has a very fast relaxation (de-excitation) capability, used for laser mode locking.

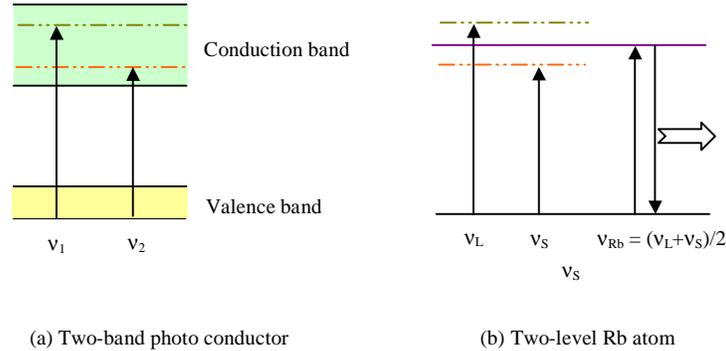


Fig. 2. (a) Shows that a photo-conductor can respond simultaneously to both the frequencies, ν_L and ν_S , because of their broad energy bands. (b) Represents one of the Rb fine structure lines. The Rb atomic dipoles can neither respond to ν_L or ν_S because they do not match with the sharp ν_{Rb} frequency, nor can they respond to $(\nu_L + \nu_S)/2$ because the fields, by themselves, do not reorganize themselves as a new, mean frequency, even though $\nu_{Rb} = (\nu_L + \nu_S)/2$. To excite Rb, one needs an EM field with instantaneous frequency exactly equaling ν_{Rb} .

2.2 Experiment 2. A high-resolution spectrometer observes both ν_L and ν_S , but not the mean frequency, $(\nu_L + \nu_S)/2$.

We sent the superposed beams through a scanning, high resolution, Fabry-Perot spectrometer (Burleigh Model RC-100). The result showed the independent presence of both the original frequencies ν_L and ν_S , (i.e., ν_1 and ν_2) as shown by the LHS of Eq. (2), but not the mean frequency, $(\nu_L + \nu_S)/2$ (or, $\nu_m = (\nu_1 + \nu_2)/2$), as shown by the RHS of Eq. (2). Further, the high-speed detector could not detect any oscillation in the intensities of any one of the resolved lines at the beat frequency $(\nu_L - \nu_S)$; they remained CW. We conclude that by simple superposition, EM fields neither generate any new mean frequency, nor does it make any permanent “mode locked” intensity modulation (beat frequency) of the EM field.

2.3 Experiment 3. Rb gas does not resonate to the mean frequency even when $\nu_{Rb} = (\nu_L + \nu_S)/2$.

We tuned ν_L and ν_S such that $(\nu_L + \nu_S)/2$ matches exactly with one of the Rb fine structure lines. We observed no appreciable resonance fluorescence (see Fig. 3(a), (b) and (c)). But strong resonances were observed whenever the laser was tuned such that either ν_L or ν_S directly matched with any one of the ν_{Rb} fine structure lines (see Fig. 2(b)). This is the second experimental validation that EM fields of different frequencies do not interact with each other. [The line-width of ν_L from the external cavity laser was well below 1MHz. Since the Doppler broadening in Rb gas is about 1.5GHz, we chose an AOM to generate ν_S with approximately 2 GHz shift, such that $(\nu_L - \nu_S) > 1.5\text{GHz}$].

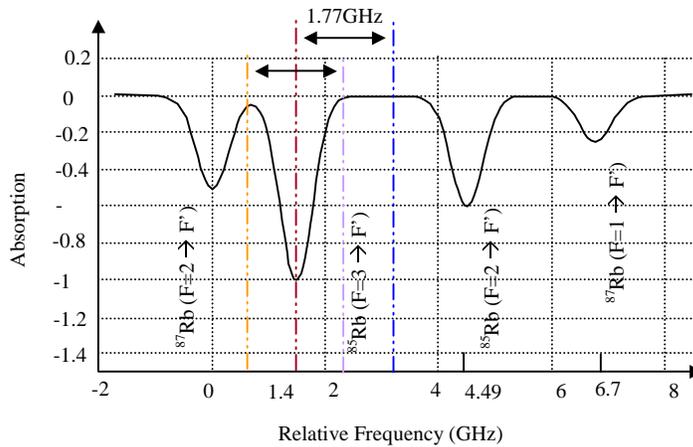


Fig. 3(a). Doppler broadened resonance fluorescence fine structure of two Rb isotopes [14, 15].

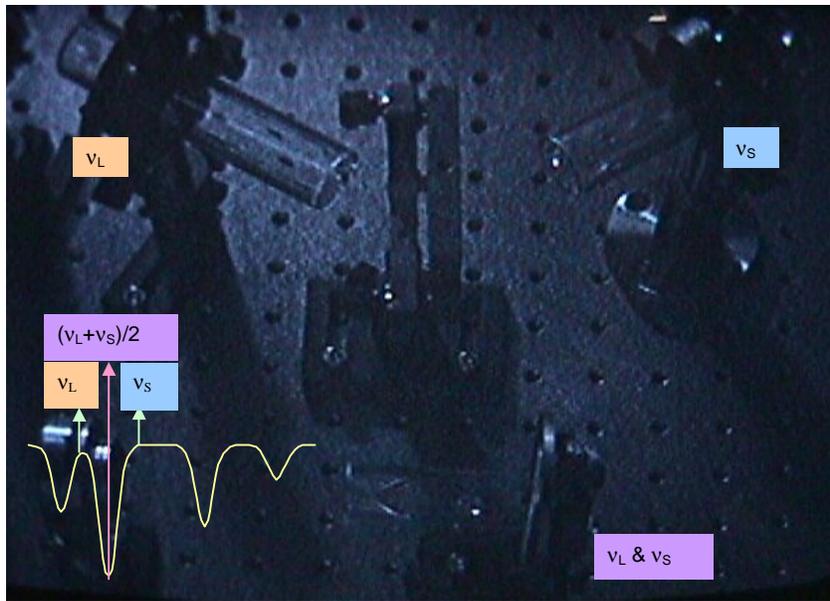


Fig. 3(b). This photograph shows three Rb tubes as sketched in the yellow shaded box of Fig. 1. The frequencies ν_L and ν_S were set such that $\nu_{Rb} = (\nu_L + \nu_S)/2$ matched the strongest line of Rb as shown in the spectral curve at the left-bottom. Notice that the Rb tube at the left top shows a weak fluorescence, as expected from the spectral curve above. The absence of fluorescence from the top-right tube is also predictable from the spectral curve. But, the important result is the absence of strong fluorescence at the bottom-center tube because superposed ν_L and ν_S did not become $\nu_{Rb} = (\nu_L + \nu_S)/2$.

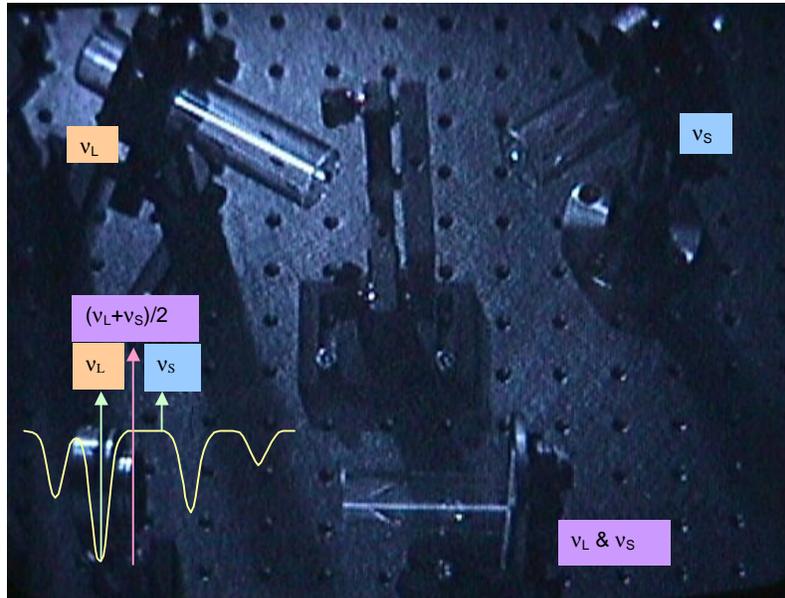


Fig. 3(c). This photograph shows strong resonance fluorescence by Rb atoms in the left-top cell because ν_L was sharply tuned to one of the Rb line center. Note the Doppler broadened spectral curve at left-bottom. The right-top tube showed no activity since ν_S did not match any of the Rb-lines. If the superposed ν_L, ν_S had become $(\nu_L + \nu_S)/2$, the Rb cell at the bottom-center should not have fluoresced. The visible, weak fluorescence is because of the presence of resonant ν_L with weakened intensity due to energy loss at the beam splitter, BS2 (see Fig. 1).

2.4 Experiment 4. A high-speed detector cannot observe beat signal ($\nu_1 - \nu_2$) when the two beams are orthogonally polarized.

Next we attempted to measure the beat frequency again, but this time, after deliberately rotating the state of polarizations of the superposed beams to be orthogonal to each other. As understandable, there was no beat signal. Light beams of orthogonal polarizations do not interfere with each other in a linear medium; it is a common knowledge [12, 13]. However, such a statement usually implies light beams of same frequency. We are considering here light beams of different frequencies. And, our explanation is that the same set of dipoles in a material, even with broad absorption bands, cannot simultaneously oscillate in two orthogonal directions. Again, this underscores our assertion that superposition effect is manifests in materials, not in the field.

3. Understanding the experiments

The observations of the above four experiments are neither surprising, nor novel, when taken separately. However, when taken together, the only conceptually self-consistent model that emerges, can be summarized as follows: (i) First, the EM fields of different frequencies do not interfere with each other or modify their field properties by themselves. This is also the key principle behind the very successful experimental field of Fourier transform spectroscopy, first demonstrated over a century ago by Michelson [16]. So, we are not adding any new understanding. (ii) Second, the principle of superposition manifests only in interacting materials and the observed results reflect the quantum properties of the constituent dipoles. Another way of saying it is that the interference effect is a collective effort by the dipoles of a material to respond simultaneously to all the superposed EM fields of different frequencies existing at their physical site. This superposition effect manifests, or becomes quantum mechanically allowed when,

$$h\nu_p \leq \Delta E_{\text{bands}}, \quad (4)$$

where, ΔE_{bands} represents the maximum energy differential between the neighboring, excitable bands of the material, and ν_p represents all possible E-vectors that can simultaneously induce dipole oscillations in the material [see Fig. 2(a)]. This is also not a new understanding. All semi-classical approaches to light-matter interaction follow this route. However, the value of this paper comes from our underscoring that matter has to carry out the operation of summation locally for the superposition effects to be manifest. Photons or EM fields do not have to have any non-local knowledge of the apparatus facilitating the superposition.

During the photoconductive detection process, electrons are excited from the valence band to the conduction band of a fast solid-state detector. If more than one coherent optical frequency, but of same polarization, is incident on the detector, the dipoles try to oscillate collectively and simultaneously at all the allowed frequencies ν_p , effectively carrying out the summation operation given by the Eqs. (1) and (2). This leads to time varying dipole excitation and consequent time varying energy exchange between the EM field and the detector. This time varying photoconductivity oscillates at the beat frequency given by Eq. (3). It is to be noted that the beat oscillation did not exist in the free EM field as demonstrated by the high-resolution spectroscopy experiments.

In the case of Rb gas, although the atoms collectively have a Doppler broadened energy band of about 1.5GHz, the energy levels of each independent atom are very sharp [see Fig. 2(b) and Fig. 3(a)]. Thus to excite any Rb-atom dipole, the incident light beam must have a precisely matching optical frequency ν_{Rb} (albeit Doppler shifted) so that the energy difference between the sharp atomic levels is exactly $\Delta E = h\nu_{\text{Rb}}$. Rb atomic dipoles cannot respond to the mean frequency of Eq. (2) even when it arithmetically matches to $\nu_{\text{Rb}} = (\nu_L + \nu_S)/2$, because EM fields did not interfere by themselves to generate this new, mean frequency.

It is very instructive to discuss the case of laser mode locking of Eq. (1) here, because it provides both contradiction and conceptual harmony. There is a contradiction because, unlike the beat signal, mode locked pulses do exist as time pulses of EM fields. There is conceptual harmony because the pulses are generated due to the superposition effect in the material (modulated gain medium, saturable absorber (SA), etc.), and not by the free fields themselves. (See Siegman [3] for various types of mode locking techniques.) The two-level band diagram for a photo-conductor, as depicted in Fig. 2(a), can also represent the dipoles of an SA, except that the excited electrons in the upper band rapidly relaxes back to the lower band. It functions like a time domain, on-off gate by responding to the in-phase (mode locked), superposed light carrying all the frequencies ν_p and absorbing energy from the field given by the time averaged, square modulus of Eq. (1). Spontaneous emission in a laser cavity starts opening the SA gate to the cavity mirror, allowing weak feedback to establish the longitudinal modes of the cavity. The desired cascading effects start due to a fortunate, statistical phase matching of the complex amplitudes of different longitudinal modes (frequencies) in the SA, making it more transparent and enhancing the gain further. Even though the field superposition is linear in the amplitudes, the energy exchange is probabilistic and quadratic $\langle |E(t)|^2 \rangle$, creating a natural preference for in-phase oscillation (higher gain due to quadratically higher transparency of the SA). Our model predicts that an appropriate spectral analysis of truly *mode locked* pulses should always reveal the existence of all the cavity longitudinal modes allowed by the gain envelope, since the EM fields do not rearrange themselves to a new, mean E-vector frequency.

4. Discussion

Light does not interfere with light! In any linear, transparent medium, light beams cross each other without modifying their frequency, or redistributing their energy (see p. 738 of Ref. [12]). We have only re-validated this through a set of four experiments. However, many important and subtle interpretations can be derived in the process of finding conceptual continuity amongst these experiments. We will briefly underscore only three of them here. (i)

Ascribing the effects of interference exclusively on the field entities will naturally perpetuate conceptual conflicts, like “non locality of interference”, “delayed choice”, “quantum many worlds”, etc., elaborated in the Refs. [17-19]. Instead of interpreting a dark fringe (in time and/or space) due to non-arrival of photons (assumption - fields interfere by themselves), we should recognize that local dipoles could not be excited (and hence, could not absorb energy) when the resultant amplitude of the superposed fields in the vicinity of the detecting dipole is zero. Thus, we claim that interference is causal and local. (ii) The concepts of Fourier synthesis and decomposition, represented by Eqs. (1) and (2), going from left-to-right and from right-to-left, respectively, are mathematical embodiment of the principle of superposition. Unfortunately, extra confusion comes in because the solutions for Maxwell’s equations and the basis functions for Fourier analysis are both sinusoidal. This paper has focused on Fourier synthesis, going from left-to-right of Eqs. (1) and (2). We have demonstrated that these equations, while mathematically right, do not represent reality of nature generally, but only conditionally, when the right dipoles interact with the fields. Consequently, by mathematical self-consistency argument, Fourier decomposition (going from right-to-left) cannot also be a generalized principle of physics. In other words, Fourier transformed mathematical frequencies of an amplitude modulated light beam could not represent real E-vectors in any linear medium. This point has been separately elaborated in Ref. [20]. Only excited dipoles can generate new EM frequencies. Finally, (iii) we must caution against indiscriminate assignment of observed quantum properties of material dipoles onto the EM fields, because fields do not operate on each other, only dipoles do. Further developments will be presented in the follow up papers.

Acknowledgements

We would like to acknowledge partial support for these research activities by the NSG, America. We would also like to thank Winthrop Smith for carefully reading the manuscript.