Holographic Pomeron and Schwinger Mechanism

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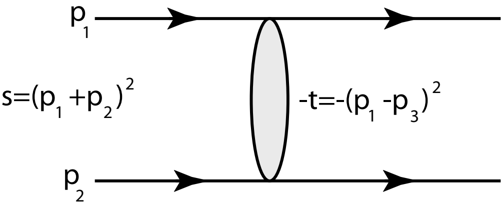
Outline

- Motivation and basics
- Calculation of the scattering amplitude
- Schwinger string production
- Conclusions

Motivation



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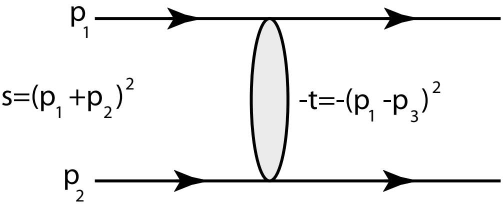

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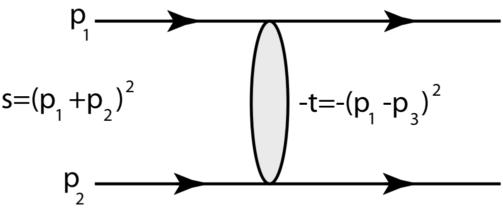

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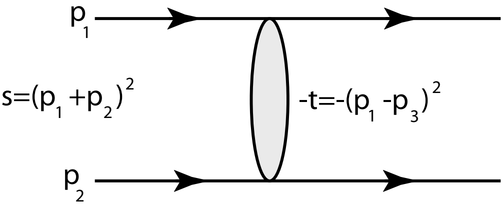

- Understand near forward, high energy ($s >> -t$) hadronic scattering amplitudes
- Small momentum transfer $\rightarrow$ non-pertrubative
- Large impact parameter $\rightarrow$ confinement
- Total cross section: $\sigma_{tot}(s) = s^{-1}\mathcal{I}m\left[\mathcal{T}(s, t=0)\right]$

Regge theory

Analyticity and crossing symmetry of the S matrix

$$\mathcal{T}(s,t) \approx \beta(t)\frac{s^{\alpha(t)} \pm (-s)^{\alpha(t)}}{\sin(\pi\alpha(t))}$$

Singularities of $\mathcal{T}$ in the complex angular momentum plane

$\Updownarrow$

Exchange of families of states in t-channel ("Reggeons")

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Regge trajectory: $\alpha(t)$

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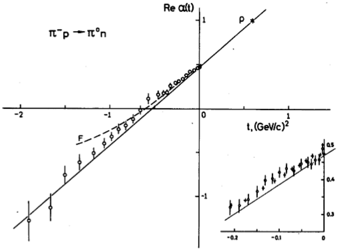
Regge trajectory: $\alpha(t) = \alpha(0) + \alpha' t$



(Apel et al. 79)

Pomeron

$\sigma_{tot} \propto s^{\alpha(0)-1}$

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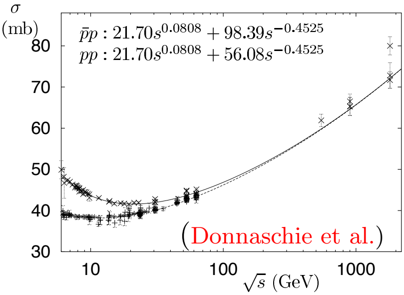

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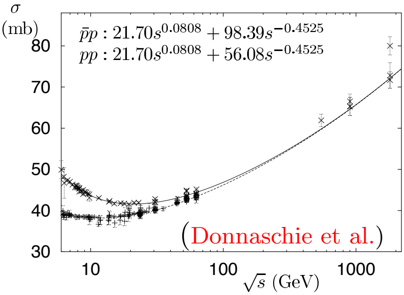
Pomeron: Regge trajectory with $\alpha_{\mathbb{P}}(0) \approx 1$



(Donnaschie et al.)

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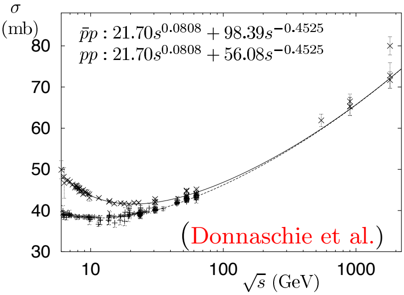

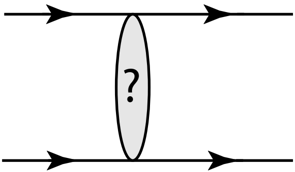

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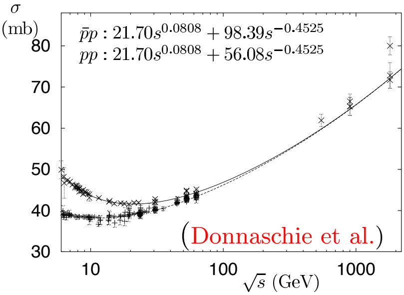
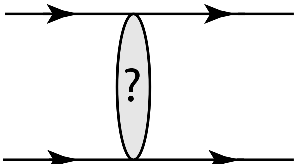
$\sigma_{tot} \propto s^{\alpha(0)-1}$ Pomeron: Regge trajectory with $\alpha_{\mathbb{P}}(0) \approx 1$

- (Donnachie, Landshoff), fit to experiment, $\alpha(0) \approx 1.08$
- (Low-Nussinov), two gluon exchange, $\alpha(0) = 1$
- (BFKL), gluon ladder, (cylinder topology at large N_c), $\alpha(0) \approx 1.3$
- ...





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Calculate dipole-dipole amplitude through gauge/string duality

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$$\text{Dipole} \sim \text{Wilson loop}$$

Wilson lines

$$\mathbf{W}(\mathcal{C}) = \frac{1}{N_c}\mathrm{Tr}\left[\mathbf{P}_c\exp\left(ig\int_{\mathcal{C}} A_\mu(x)\, dx^\mu\right)\right]$$

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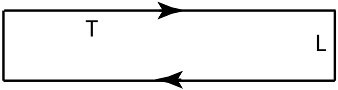
Static Wilson loop:



$< \mathbf{W} > \approx e^{-V(L)\,T}$ $\quad V(L)$: potential between 2 static charges

- Abelian gauge field: $V(L) \propto 1/L \Rightarrow$ "Perimeter law"
- Confining potential: $V(L) = \sigma_T\, L \Rightarrow$ "Area law"

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High energy scattering:

- Recoil of scatterers is negligible $\rightarrow$ follow fixed trajectories
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  gluon ∼ color dipole at large N_c

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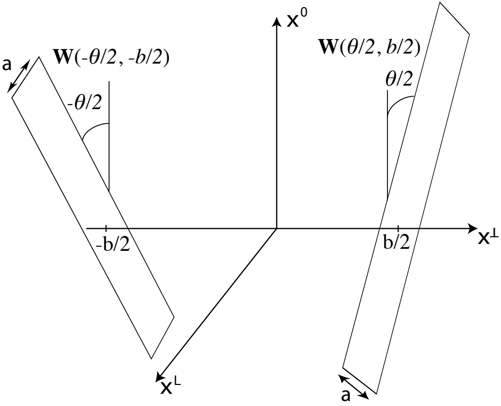
high energy hadron ∼ gluon cloud ∼ bunch of dipoles
gluon ∼ color dipole at large N_c

Dipole-dipole kinematics

$$D_1\,(p_1) + D_2\,(p_2) \rightarrow D_1\,(k_1) + D_2\,(k_2)$$



$p_1/m = (\cos(\theta/2), -\sin(\theta/2), 0^{\perp})$

$p_2/m = (\cos(\theta/2), \sin(\theta/2), 0^{\perp})$

$q = (0,0,q^{\perp})$

$b = (0,0,b^{\perp})$

$\theta \longrightarrow -i\chi$

$\chi \approx \log(s/2m^2) >> 1$

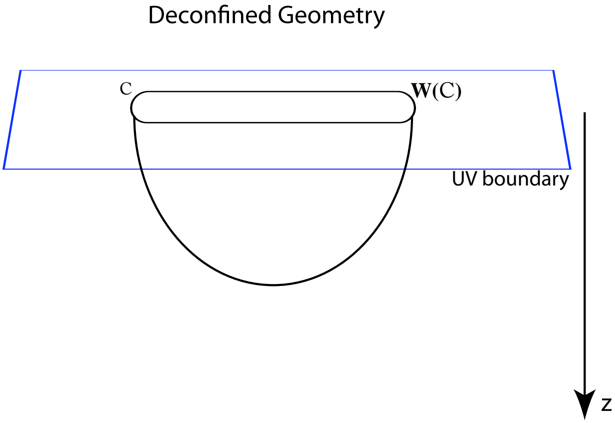
(Meggiolaro, Giordano, Janik, Peschanski, Nowak, Shuryak, Zahed...)

(Nachtmann, '91)

$$\frac{1}{-2is}\mathcal{T}(\theta,q) \approx \int d^2b\; e^{iq_\perp \cdot b}\langle(\mathbf{W}(\theta/2,b/2)\mathbf{W}(-\theta/2,-b/2)-\mathbf{1})\rangle$$

Wilson lines in holography

$$< \mathbf{WW} >= Z_{string}[\partial\mathcal{B} = \mathcal{C}]$$ (Maldacena, '98)



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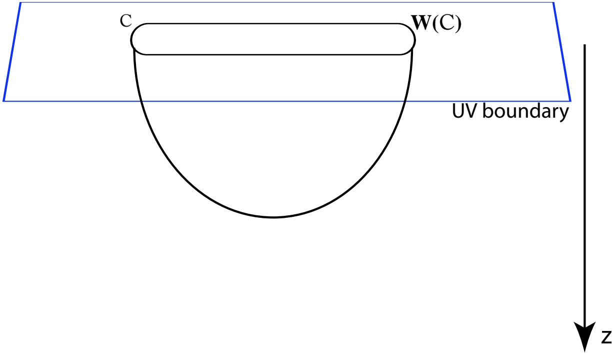
Deconfined Geometry



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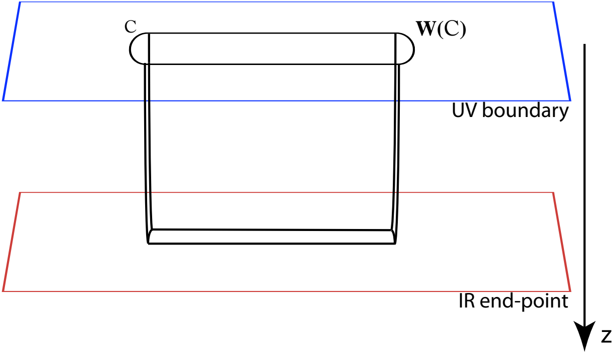

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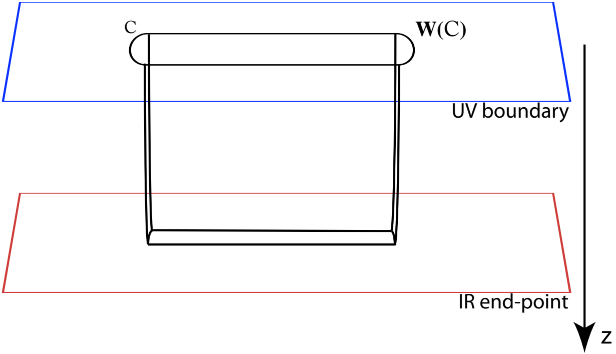

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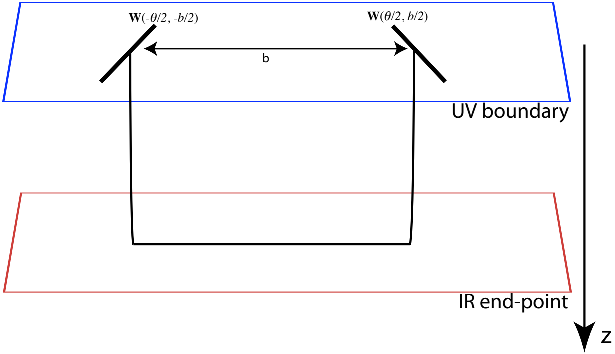

String partition function

1-loop (cylinder) partition function:

$$< \mathbf{WW} >= g_s^2 \int_0^\infty \frac{dT}{2T} \int_{\mathrm{T}} d[x]\, e^{-\mathbf{S}[x]+\text{ghosts}} = g_s^2 \int_0^\infty \frac{dT}{2T} \mathbf{K}(T)$$

$$\mathbf{S} = \frac{\sigma_T}{2} \int_0^T d\tau \int_0^1 d\sigma \left(\dot{x}^\mu \dot{x}_\mu + x'^\mu x'_\mu \right)$$

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Boundary conditions:

$$time: \qquad x^\mu(T,\sigma) = x^\mu(0,\sigma)$$

$$space: \qquad \begin{aligned} &\cos(\theta/2)\, x^1(\tau,0) + \sin(\theta/2)\, x^0(\tau,0) = 0 \\ &\cos(\theta/2)\, x^1(\tau,1) - \sin(\theta/2)\, x^0(\tau,1) = 0 \end{aligned}$$

String partition function

$$\mathbf{K}(T) = \frac{\pi\sigma_T\, a^2}{\sinh(\theta T/2)}\; e^{-\sigma_T b^2 T/2}\; \eta^{-D_\perp}(iT/2) \prod_{n=1}^{\infty}\prod_{s=\pm} \frac{\sinh\left(\pi n T/2\right)}{\sinh(\pi(n+s\theta/\pi)T/2)}$$

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$\frac{\sigma_T}{2}\, b^2 T$: classical worldsheet action

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$\frac{1}{\sinh(\theta \mathrm{T}/2)} \prod_{n=1}^{\infty}\prod_{s=\pm} \frac{\sinh(\pi n T/2)}{\sinh(\pi(n+s\theta/\pi)T/2)}$: longitudinal modes

Back to Minkowski space: $\theta \to -i\chi$

$$< \mathbf{WW} >= \frac{ia^2 g_s^2}{4\alpha'} \int_0^\infty \frac{dT}{T} \frac{1}{\sin(\chi T/2)} \, e^{-\sigma_T b^2 T/2} \, \eta^{-D_\perp}(iT/2) \prod(..)$$

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Tunneling amplitude:

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Multi-wrapping: k≡"N-ality" , $k > 1$: N_c suppressed

Leading pole:

$$< \mathbf{WW} >= -a^2 \pi g_s^2 (4\pi\alpha')^{D_\perp - 1} \left(\frac{1}{2\pi\alpha'\chi}\right)^{D_\perp/2} e^{-b^2/2\alpha'\chi + D_\perp \chi/12}$$

Diffusion term

$$< \mathbf{WW} >= -a^2\pi g_s^2(4\pi\alpha')^{D_\perp - 1}\left(\frac{1}{2\pi\alpha'\,\chi}\right)^{D_\perp/2} e^{-b^2/2\alpha'\chi + D_\perp\chi/12}$$

Diffusion in transverse space (Gribov '75)

$$\mathbf{K}(\chi, b) = \left(\frac{1}{2\pi\alpha'\,\chi}\right)^{D_\perp/2} e^{-b^2/2\chi\alpha'}$$

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Rapidity $\chi \leftrightarrow$ "diffusion time" $\alpha' \leftrightarrow$ diffusion constant

Lüscher term and the intercept

$$< \mathbf{WW} >= -a^2 \pi g_s^2 (4\pi\alpha')^{D_\perp - 1} \left(\frac{1}{2\pi\alpha' \chi} \right)^{D_\perp/2} e^{-b^2/2\alpha'\chi + D_\perp \chi/12}$$

Lüscher term (Casimir energy):

$$\frac{D_\perp \chi}{12} = \frac{\pi D_\perp}{6} \frac{b}{T} = b \cdot V(T) \quad , \quad T = \frac{2\pi b}{\chi}$$

$$e^{\frac{D_\perp \chi}{12}} = s^{D\perp/12} \text{ (intercept)}$$

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Fermions?

$$V(T)_F \sim D_\perp \sqrt{\frac{M_{KK}}{T}} e^{-2M_{KK}T} \qquad \text{(Sonnenschein et al., '00)}$$

Scattering amplitude and cross section

$$\mathcal{T}(s,t) = -2is \int d^2b\ e^{iq_\perp b} < (\mathbf{WW} - 1) >$$
$$= ia^2\pi^2 g_s^2 \left(\frac{\pi}{\ln s}\right)^{D_\perp/2-1} s^{\alpha(t)}$$

Regge trajectory: $\alpha(t) = 1 + \frac{D_\perp}{12} + \frac{\alpha'}{2}t$

Scattering amplitude and cross section

$$
\begin{aligned}
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$$

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Total cross section: $\sigma_{tot} = a^2\pi^2 g_s^2 \left(\frac{\pi}{\ln s}\right)^{D_\perp/2-1} s^{D_\perp/12}$

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- Saturates the Froissart bound

Schwinger mechanism and Regge trajectory

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Intermezzo: Schwinger pair production

Worldline formalism (Dunne,Schubert):

$$\Gamma[A] \;=\; \int_0^\infty \frac{dT}{T} e^{-m^2\,T} \int_{PBC} d[x]\, exp\left[-\int_0^T d\tau(\frac{\dot{x}^2}{4} + iA\cdot\dot{x})\right]$$

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action: $S_{cl} = k\frac{\pi m^2}{eE}$, rate: $\Gamma \sim e^{-k\frac{\pi m^2}{eE}}$

Schwinger string production

$$\Gamma_s = g_s^2 \int_0^\infty \frac{dT}{2T} \int_{PBC} d[x]\, exp\left[-\int_0^T d\tau \int_0^1 d\sigma (\dot{x}^2 + x'^2) \right]$$

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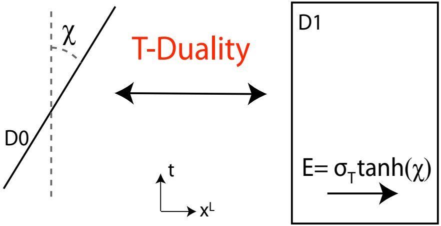

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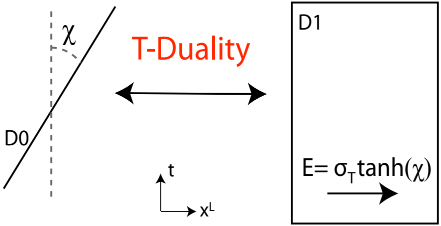

twisted b.c. $\Leftrightarrow$ $\frac{E}{2} \int d\tau \left(\tilde{x}^1 \partial_\tau x^0 - x^0 \partial_\tau \tilde{x}^1 \right) \Big|_{\sigma=0,1}$

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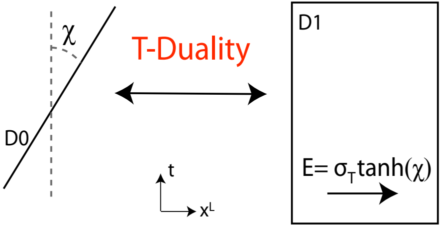

twisted b.c. $\quad \Leftrightarrow \quad \int d\tau \left(iA \cdot \dot{\tilde{x}} \right) \Big|_{\sigma=0,1}$

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"Worldsheet instantons" (Schubert):

$$x^0 = R(\sigma)\cos(2\pi k u)\ \ ,\ \ \tilde{x}^1 = R(\sigma)\sin(2\pi k u)\ \ ,\ \ x^T = b\,(\sigma - 1/2)$$

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(Bachas, Porrati '92)

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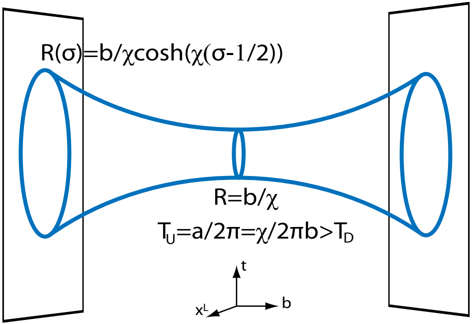
Electric field $\leftrightarrow$ Acceleration $\leftrightarrow$ Unruh effect



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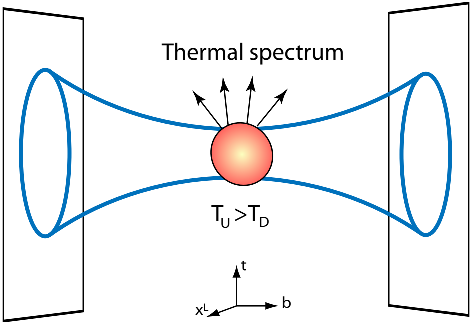

Conclusions:

- nonperturbative approach to soft pomeron (slope and intercept)
- inelasticity & Regge behavior $\Leftrightarrow$ string creation à la Schwinger
- fireball in the center of the collision with thermal radiation
- relative rapidity $\chi \Leftrightarrow$ electric field $E_L = \sigma_T \tanh\chi$