Some Mathcad Examples

Prepared for Physics 258 by Ed Eyler, September 1999.

Let's start by entering an integral to be evaluated numerically, similar to the one encountered in the
large-amplitude pendulum lab. We will talk more about the techniques used for integration later in the course, when you will have a chance to program some integrals for yourself. The default tolerance for numerical evaluations is 0.001 (you can modify this), so the last displayed decimal places may or may not be accurate.

$k := 2.1$ Define variables before using them, unless they are global defini- tions (use a tilda to define globals--they are known everywhere.)

Getting right to the point, we write,

$$\frac{2}{3}\cdot\int_{0}^{\frac{\pi}{2}} \frac{1}{\sqrt{1 + k^2\cdot\tan(x)}}\,dx = 0.47523$$

Now use a range variable to evaluate the same integral for a variety of values of k. Note that the range variable must be an integer if we're going to use it as a subscript for indexing.

$$\alpha := 0, 2 .. 50$$

$$k_{\alpha} := \cos\left(\frac{\alpha\cdot\pi}{180}\right)$$ (Radians are the default angular measure)

$$f_{\alpha} := \frac{2}{3}\cdot\left[\int_{0}^{\frac{\pi}{2}} \frac{1}{\sqrt{1 + \left(k_{\alpha}\right)^2\cdot\tan(x)}}\,dx\right]$$

Display the results, with a little variety in formatting to show what's available:

α =

	0
0	0
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18

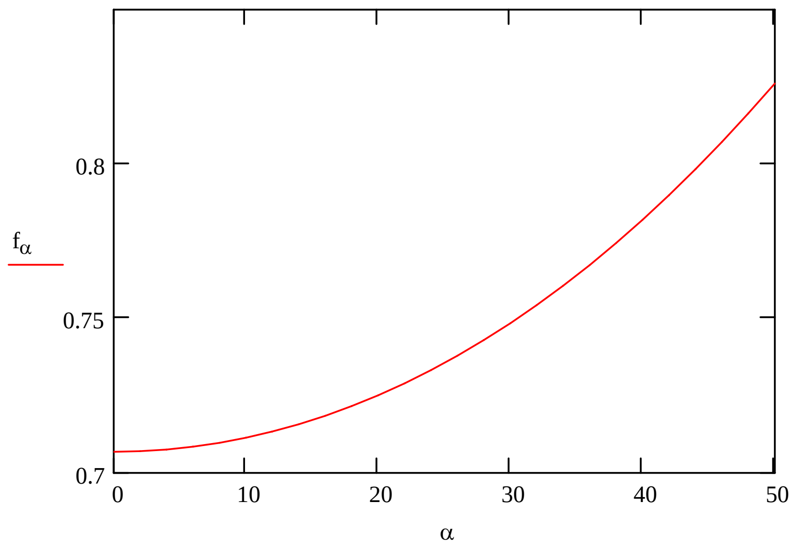
$_{\alpha}$ =

1
0.999
0.998
0.995
0.99
0.985
0.978
0.97
0.961
0.951

f_{α} =

0.7068224727
0.7070033675
0.7075462772
0.7084518766
0.7097212868
0.7113560694
0.7133582182
0.7157301482
0.7184746813
0.7215950277
0.7250947633
0.7289778014
0.7332483582

Mathcad has many, many plotting options. Use a simple x-y graph to examine the integral:

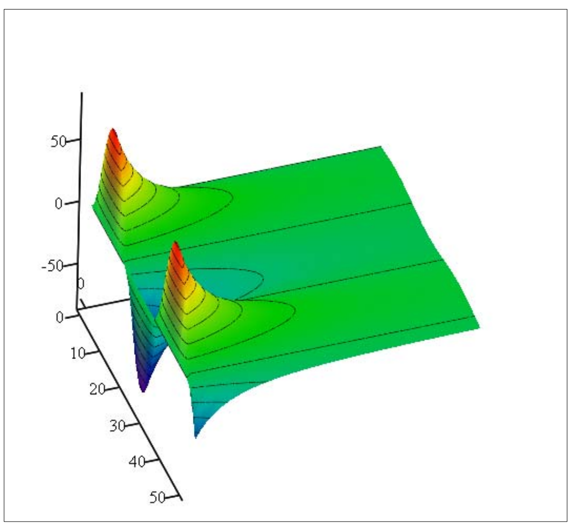


You can also do fancier stuff: Use the mouse to change perspective for this plot:

$j := 1..50$

$k := 1..50$

$$M_{j,k} := \frac{1 \cdot 10^4 \cdot \sin\left(\frac{j}{5}\right)}{(k+10)^2}$$



M

You can obtain input data either by typing it directly or by reading from files with Import (one-time) or the "File Read or Write" component (allows for updates). Let's define an input table using Insert, Component, Input Table:

mydata :=

	0	1
0	0	0
1	2	4.1
2	4	15.5
3	6	35
4	8	63
5	12	142
6	14	197
7	16	255
8	20	390
9		
10		
11		

Because we entered an n X 2 matrix, we can treat it with matrix operators if we wish. For

$$\text{mydata}\cdot\text{mydata}^{T} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20.81 & 71.55 & 155.5 & 274.3 & 606.2 & 835.7 & 1.077\times10^{3} \\ 0 & 71.55 & 256.25 & 566.5 & 1.008\times10^{3} & 2.249\times10^{3} & 3.11\times10^{3} & 4.016\times10^{3} \\ 0 & 155.5 & 566.5 & 1.261\times10^{3} & 2.253\times10^{3} & 5.042\times10^{3} & 6.979\times10^{3} & 9.021\times10^{3} \\ 0 & 274.3 & 1.008\times10^{3} & 2.253\times10^{3} & 4.033\times10^{3} & 9.042\times10^{3} & 1.252\times10^{4} & 1.619\times10^{4} \\ 0 & 606.2 & 2.249\times10^{3} & 5.042\times10^{3} & 9.042\times10^{3} & 2.031\times10^{4} & 2.814\times10^{4} & 3.64\times10^{4} \\ 0 & 835.7 & 3.11\times10^{3} & 6.979\times10^{3} & 1.252\times10^{4} & 2.814\times10^{4} & 3.901\times10^{4} & 5.046\times10^{4} \\ 0 & 1.077\times10^{3} & 4.016\times10^{3} & 9.021\times10^{3} & 1.619\times10^{4} & 3.64\times10^{4} & 5.046\times10^{4} & 6.528\times10^{4} \\ 0 & 1.639\times10^{3} & 6.125\times10^{3} & 1.377\times10^{4} & 2.473\times10^{4} & 5.562\times10^{4} & 7.711\times10^{4} & 9.977\times10^{4} \end{pmatrix}$$

We can easily find the mean of all the data in the matrix,

$$\text{mean}(\text{mydata}) = 65.756$$

Or of the data in column 1:

$$\text{mean}\left(\text{mydata}^{\langle 1\rangle}\right) = 122.4$$

We can fit data using a simple linear least-squares fit, a polynomial regression, or a generalized least-squares fit to an arbitrary function. The data above are obviously close to quadratic, so let's try a second-order regression. (If we were sure there's no linear component, we would probably instead take the square root and fit it to a straight line.) Anyhow, we can use the column operators to define the x and y data. To be explicit we define vectors,

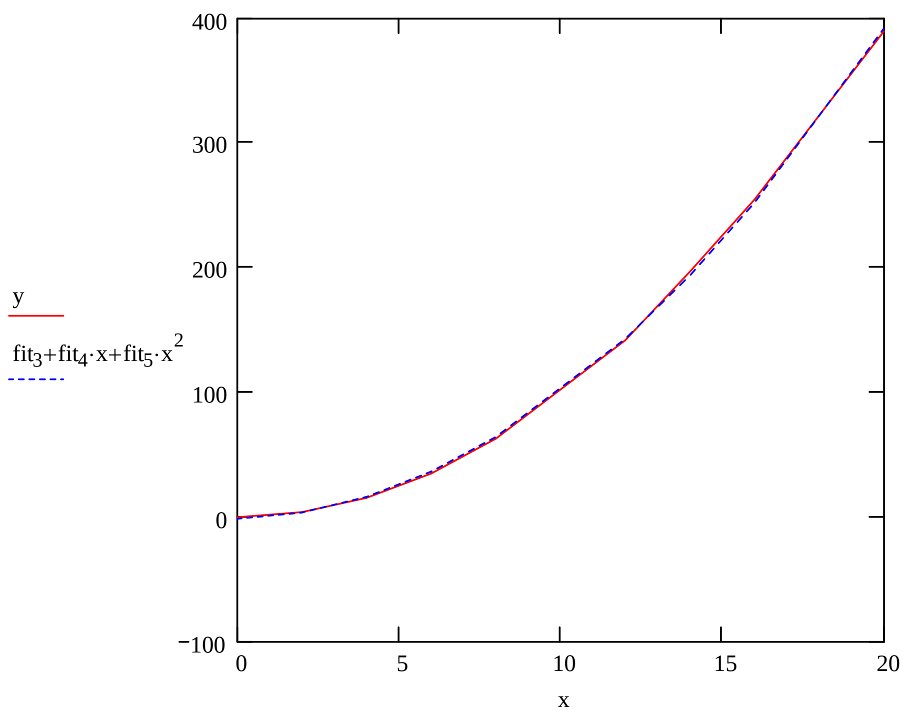
$x := mydata^{\langle 0 \rangle}$

$y := mydata^{\langle 1 \rangle}$

$$x = \begin{pmatrix} 0 \\ 2 \\ 4 \\ 6 \\ 8 \\ 12 \\ 14 \\ 16 \\ 20 \end{pmatrix}$$

$fit := regress(x, y, 2)$

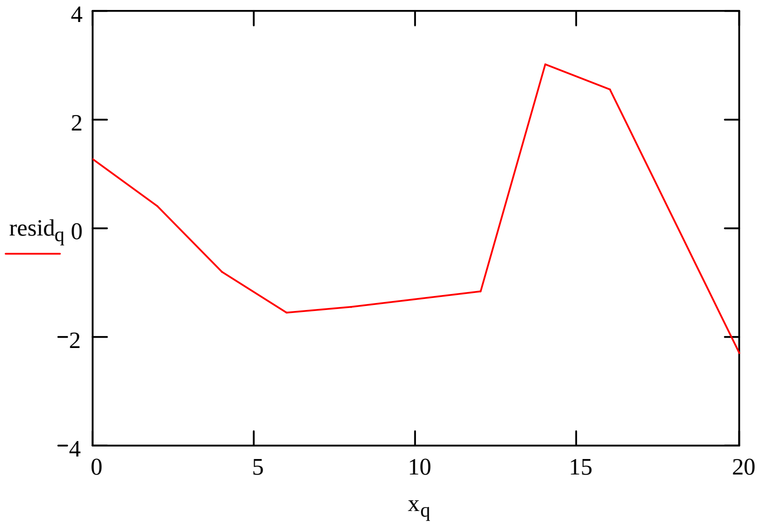
$$fit = \begin{pmatrix} 3 \\ 3 \\ 2 \\ -1.275 \\ 0.573 \\ 0.955 \end{pmatrix}$$



Now take a look at the fit residuals:

$q := 0 .. rows(x) - 1$

$resid_q := y_q - interp(fit, x, y, x_q)$ (Interp gives the same result as subtracting the fitting function explicitly, but also can interpolate in between the data points if desired.)



We cannot quite evaluate the value of chi-squared, since we don't know the standard deviation for the measurements. However, we can measure the mean-squared deviation, which reflects the quality of the fit just as well, if all of the data points have the same uncertainty:

$$msd := \frac{\sum_q (resid_q)^2}{rows(x)}$$

$$msd = 3.245$$

The least-squares fit in the regression function operates by minimizing this quantity. For example, if we try a very slightly different set of fit parameters, redefining the residuals to be

$$resid_q := y_q - \left[fit_3 + fit_4 \cdot x_q + (fit_5 - 0.005) \cdot (x_q)^2\right]$$

we find that the mean-squared deviation increases a little bit,

$$\frac{\sum_q (resid_q)^2}{rows(x)} = 4.051$$

Other MathCad capabilities include root finding, solutions of differential equations, numerous vector operations and special functions, and the ability to treat dimensioned numbers. Complex numbers are handled transparently, without any change in notation:

$$\operatorname{acos}(17) = 3.525i$$

$$\sqrt{-20} = 4.472i$$

$$\exp(1 + 2i) = -1.131 + 2.472i$$

Finally, MathCad has a limited subset of Maple, allowing some symbolic ability:

$(x + 2\cdot a)^6$ expands to give

$$x^6 + 12\cdot x^5\cdot a + 60\cdot x^4\cdot a^2 + 160\cdot x^3\cdot a^3 + 240\cdot x^2\cdot a^4 + 192\cdot x\cdot a^5 + 64\cdot a^6$$

$\frac{d}{dx}\sin\left(\frac{1}{\sqrt[3]{\sinh(x) + \ln(x)}}\right)$ simplifies (well, sort of) to,

$$\frac{-1}{3}\cdot\frac{\cos\left[\frac{1}{(\sinh(x) + \ln(x))^{\frac{1}{3}}}\right]}{(\sinh(x) + \ln(x))^{\frac{4}{3}}}\cdot\frac{\cosh(x)\cdot x + 1}{x}$$

$\int \frac{1}{\sin(x)}\,dx$ evaluates to

$$\ln\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

A potentially very useful feature is the ability to expand in Taylor series about an arbitrary point,

$$e^{\sin(r)^2} \text{ series}, r = \frac{\pi}{3}, 4 \rightarrow e^{\frac{3}{4}} + \left(\frac{1}{2}\cdot e^{\frac{3}{4}}\cdot 3^{\frac{1}{2}}\right)\cdot\left(r - \frac{1}{3}\cdot\pi\right) + \left(\frac{-1}{8}\cdot e^{\frac{3}{4}}\right)\cdot\left(r - \frac{1}{3}\cdot\pi\right)^2 + \left(\frac{-25}{48}\cdot e^{\frac{3}{4}}\cdot 3^{\frac{1}{2}}\right)\cdot\left(r - \frac{1}{3}\cdot\pi\right)^3$$