Physics 1502: Lecture 7
Today's Agenda

- **Announcements:**
  - Lectures posted on: www.phys.uconn.edu/~rcote/
  - HW assignments, solutions etc.
- **Homework #2:**
  - On Masterphysics today: due Friday
  - Go to masteringphysics.com
- **Labs**: Begin this week

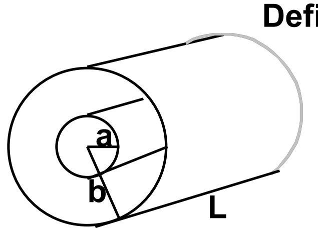
Today's Topic :

- **End Chapter 23:**
  - Definition of Capacitance
  - Example Calculations
    (1) Parallel Plate Capacitor
    (2) Cylindrical Capacitor
    (3) Isolated Sphere
  - Energy stored in capacitors
  - Dielectrics
  - Capacitors in Circuits

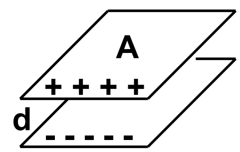
Lecture 4

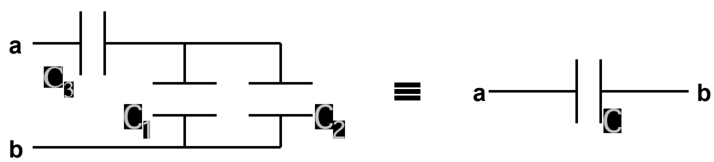
Capacitance

Definitions & Examples



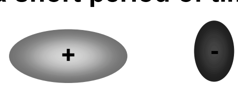
$$C \equiv \frac{Q}{V}$$





Capacitance

- A capacitor is a device whose purpose is to store electrical energy which can then be released in a controlled manner during a short period of time.

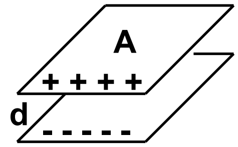


- A capacitor consists of 2 spatially separated conductors which can be charged to +Q and -Q respectively.
- The capacitance is defined as the ratio of the charge on one conductor of the capacitor to the potential difference between the conductors.

$$C \equiv \frac{Q}{V}$$

Example 1: Parallel Plate Capacitor

- Calculate the capacitance. We assume +σ, - σ charge densities on each plate with potential difference *V*:

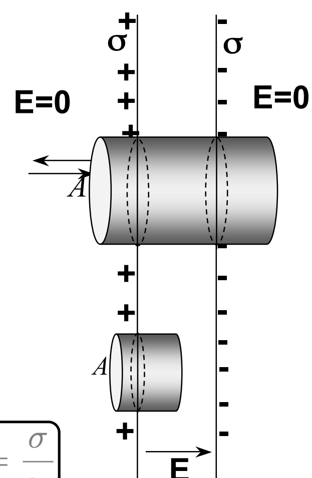


$$C \equiv \frac{Q}{V}$$

- Need Q: $Q = \sigma A$
- Need V: from defn: $V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{\ell}$
  - Use Gauss' Law to find E

Recall: Two Infinite Sheets

(into screen)



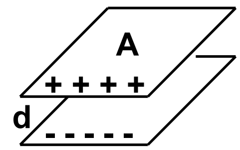
- Field outside the sheets is zero
  - Gaussian surface encloses zero net charge
- Field inside sheets is not zero:
  - Gaussian surface encloses non-zero net charge $Q = \sigma A$

$$\oint \vec{E} \cdot d\vec{S} = AE_{\text{inside}} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$

Example 1: Parallel Plate Capacitor

- **Calculate the capacitance:**

  Assume +Q,-Q on plates with potential difference V.



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

$$V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{\ell} = \frac{Q}{A\varepsilon_0} d$$

$$\Rightarrow \boxed{C \equiv \frac{Q}{V} = \frac{A\varepsilon_0}{d}}$$

- **As hoped for, the capacitance of this capacitor depends only on its geometry (A,d).**

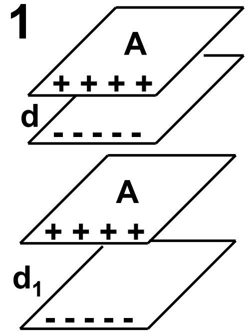
Dimensions of capacitance

- ***C* = Q/*V* => [*C*] = F(arad) = C/V = [Q/*V*]**

- **Example: Two plates, *A* = 10cm x 10cm**
  d = 1cm apart

  => $C = A\varepsilon_0/d$ =
  $= 0.01m^2/0.01m * 8.852e\text{-}12\ C^2/Jm$
  = ***8.852e-12 F***

Lecture 7 - ACT 1

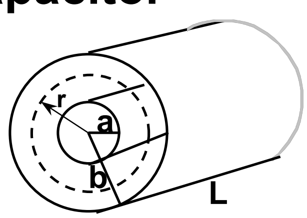


- **Suppose the capacitor shown here is charged to Q and then the battery disconnected.**
- **Now suppose I pull the plates further apart so that the final separation is d_1. $d_1 > d$**

If the initial capacitance is C_0 and the final capacitance is C_1, is …

A) $C_1 > C_0$ **B) $C_1 = C_0$** **C) $C_1 < C_0$**

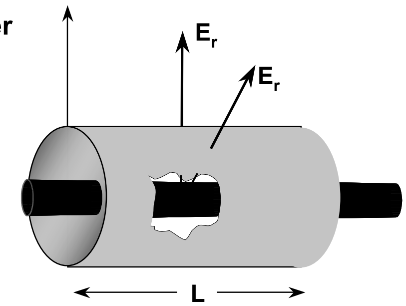
Example 2: Cylindrical Capacitor



- **Calculate the capacitance:**
- **Assume +Q, -Q on surface of cylinders with potential difference *V*.**

Recall: Cylindrical Symmetry

- Gaussian surface is cylinder of radius r and length L
- Cylinder has charge Q



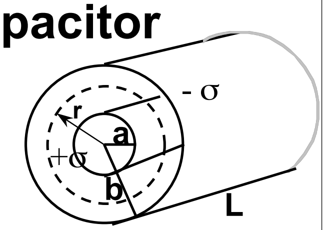
- Apply Gauss' Law:

$$\oint \vec{E} \cdot d\vec{S} = 2\pi r L E = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{Q}{2\pi\varepsilon_0 L r}}$$

Example 2: Cylindrical Capacitor

- Calculate the capacitance:
- Assume +Q, -Q on surface of cylinders with potential difference *V*.



$$E = \frac{Q}{2\pi\varepsilon_0 L r}$$

If we assume that inner cylinder has +Q, then the potential V is positive if we take the zero of potential to be defined at r = b:

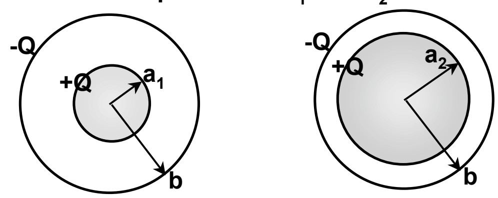
$$V = -\int_b^a \vec{E} \cdot d\vec{\ell} = -\int_b^a E\, dr = \int_a^b \frac{Q}{2\pi\varepsilon_0 L r} dr = \frac{Q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right)$$

$$\Rightarrow \boxed{C \equiv \frac{Q}{V} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}}$$

again: depends only on system parameters (i.e., geometry)

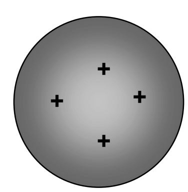
Lecture 7, ACT 2

- In each case below, a charge of +Q is placed on a solid spherical conductor and a charge of -Q is placed on a concentric conducting spherical shell.
  - Let V_1 be the potential difference between the spheres with (a_1, b).
  - Let V_2 be the potential difference between the spheres with (a_2, b).
  - What is the relationship between V_1 and V_2?



(a) $V_1 < V_2$ (b) $V_1 = V_2$ (c) $V_1 > V_2$

Example 3: Isolated Sphere



- Can we define the capacitance of a single isolated sphere ?
- The sphere has the ability to store a certain amount of charge at a given voltage (versus V=0 at infinity)

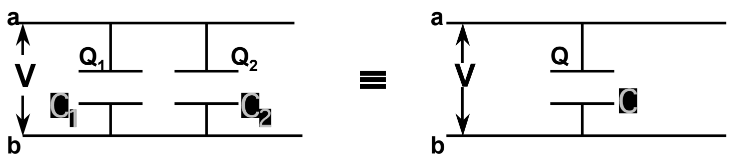
$$C \equiv \frac{Q}{\Delta V}$$

- Need ΔV: $V_\infty = 0$

  $V_R = k_e Q/R$

- So, $C = R/k_e$

Capacitors in Parallel



- **Find "equivalent" capacitance C in the sense that no measurement at a,b could distinguish the above two situations.**

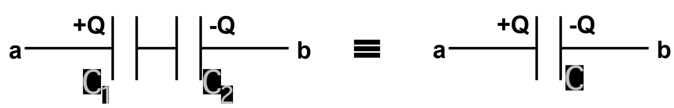
Parallel Combination: $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

=> Total charge: $Q = Q_1 + Q_2$

Equivalent Capacitor: $C \equiv \frac{Q}{V}$ [redacted]

$\Rightarrow$ $C = C_1 + C_2$

Capacitors in Series



- **Find "equivalent" capacitance C in the sense that no measurement at a,b could distinguish the above two situations.**

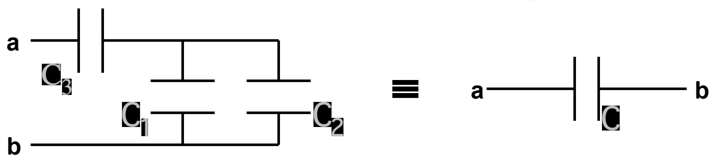
- **The charge on C_1 must be the same as the charge on C_2 since applying a potential difference across ab cannot produce a net charge on the inner plates of C_1 and C_2 .**

RHS: $V_{ab} = \frac{Q}{C}$

LHS: $V_{ab} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$

$\Rightarrow$ $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Examples: Combinations of Capacitors



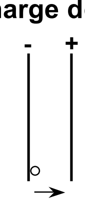
- How do we start??
  - Recognize C_3 is in series with the parallel combination on C_1 and C_2. *i.e.*

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2} \quad \Rightarrow \quad C = \frac{C_3(C_1 + C_2)}{C_1 + C_2 + C_3}$$

Energy of a Capacitor

- How much energy is stored in a charged capacitor?
  - Calculate the work provided (usually by a battery) to charge a capacitor to +/- Q:

Calculate incremental work dW needed to add charge dq to capacitor at voltage V:



$$dW = V\,dq = \left(\frac{q}{C}\right) dq$$

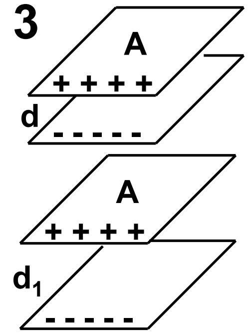
- The total work W to charge to Q is then given by:

$$W = \frac{1}{C}\int_0^Q q\,dq = \frac{1}{2}\frac{Q^2}{C}$$

- In terms of the voltage V:

$$W = \frac{1}{2}CV^2$$

Lecture 7 – ACT 3



The same capacitor as last time.

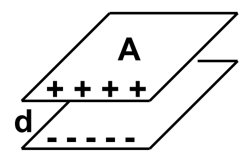
The capacitor is charged to Q and then the battery disconnected.

Then I pull the plates further apart so that the final separation is d_1. $d_1 > d$

If the initial energy is U_0 and the final capacitance is U_1, is …

A) $U_1 > U_0$ **B) $U_1 = U_0$** **C) $U_1 < U_0$**

Summary



- **Suppose the capacitor shown here is charged to Q and then the battery disconnected.**
- **Now suppose I pull the plates further apart so that the final separation is d_1.**
- **How do the quantities Q, W, C, V, E change?**
- **Q: remains the same.. no way for charge to leave.**
- **W: increases.. add energy to system by separating**
- **C: decreases.. since energy ↑, but Q remains same**
- **V: increases.. since C ↓, but Q remains same**
- **E: remains the same.. depends only on chg density**
- **How much do these quantities change?.. exercise for student!!**

answers: $W_1 = \frac{d_1}{d}W$ $C_1 = \frac{d}{d_1}C$ $V_1 = \frac{d_1}{d}V$

Where is the Energy Stored?

- **Claim: energy is stored in the Electric field itself. Think of the energy needed to charge the capacitor as being the energy needed to create the field.**
- **To calculate the energy density in the field, first consider the constant field generated by a parallel plate capacitor:**

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{(A\varepsilon_0/d)}$$

- **The Electric field is given by:**

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad \Rightarrow \quad W = \frac{1}{2}E^2\varepsilon_0 Ad$$

- **The energy density u in the field is given by:**

$$u = \frac{W}{\text{volum}} = \frac{W}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

Units: *J/m³*

Dielectrics

- **Empirical observation:**

  Inserting a non-conducting material between the plates of a capacitor changes the VALUE of the capacitance.

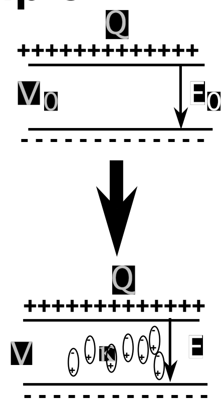
- **Definition:**

  The dielectric constant of a material is the ratio of the capacitance when filled with the dielectric to that without it. i.e.

$$\kappa = \frac{C}{C_0}$$

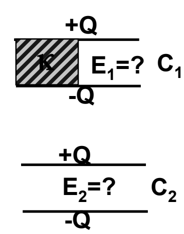
  - **κ values are always > 1 (e.g., glass = 5.6; water = 78)**
  - **They INCREASE the capacitance of a capacitor (generally good, since it is hard to make "big" capacitors**
  - **They permit more energy to be stored on a given capacitor than otherwise with vacuum (i.e., air)**

Parallel Plate Example



- Charge a parallel plate capacitor filled with vacuum (air) to potential difference V_0.
- An amount of charge $Q = C V_0$ is deposited on each plate.
- Now insert material with dielectric constant κ .
  - Charge Q remains <u>constant</u>
  - Voltage decreases from V_0 to $V = \frac{V_0}{\kappa}$
  - Electric field decreases also: $E = \frac{E_0}{\kappa}$
  - So…, $C = \kappa C_0$

Lecture 7, ACT 3



- Two parallel plate capacitors are identical (same A, same d) except that C_1 has half of the space between the plates filled with a material of dielectric constant κ as shown.
  - If both capacitors are given the same amount of charge Q, what is the relation between E_1, the electric field in the air of C_1, and E_2, the electric field in the air of C_2

(a) $E_1 < E_2$ (b) $E_1 = E_2$ (c) $E_1 > E_2$