

Lecture 4

Physics 1502: Lecture 3 Today's Agenda

- **Announcements:**
 - Lectures posted on:
www.phys.uconn.edu/~rcote/
 - HW assignments, solutions etc.
- **Homework #1:**
 - On Masterphysics today: due next Friday
 - Go to masteringphysics.com and register
 - Course ID: MPCOTE33308
- **Labs: Begin in two weeks**
- **No class Monday: Labor Day**

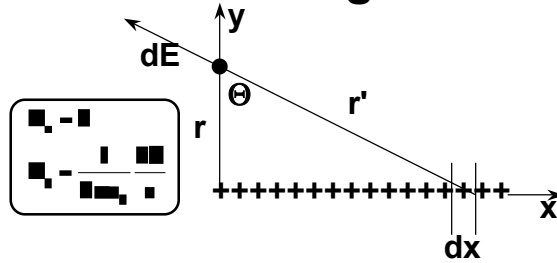
Today's Topic :

- **End of Chapter 20**
 - Continuous charge distributions => integrate
 - Moving charges: Use Newton's law
- **Chapter 21: Gauss's Law**
 - Motivation & Definition
 - Coulomb's Law as a consequence of Gauss' Law
 - Charges on Insulators:
 - » Where are they?

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Infinite Line of Charge

• Solution:



The Electric Field produced by an infinite line of charge is:

- everywhere perpendicular to the line
- is proportional to the charge density
- decreases as $1/r$.

Lecture 3, ACT 1

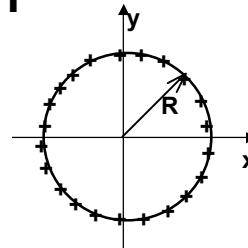
• Consider a circular ring with a uniform charge distribution (λ charge per unit length) as shown. The total charge of this ring is $+Q$.

• The electric field at the origin is

(a) zero

(b) $\frac{kQ}{R^2}$

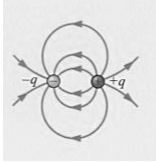
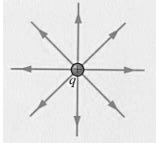
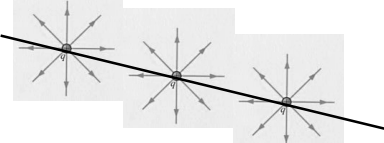
(c) $\frac{kQ}{R}$



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Summary

Electric Field Distributions

Dipole		$\sim 1 / R^3$
Point Charge		$\sim 1 / R^2$
Infinite Line of Charge		$\sim 1 / R$

Motion of Charged Particles in Electric Fields

- Remember our definition of the Electric Field,

$$\vec{E} = \frac{\vec{F}}{q}$$

- And remembering Physics 1501,

$$\vec{F} = q\vec{E}$$

Now consider particles moving in fields.

Note that for a charge moving in a constant field this is just like a particle moving near the earth's surface.

$$a_x = 0$$

$$a_y = \text{constant}$$

$$v_x = v_{ox}$$

$$v_y = v_{oy} + at$$

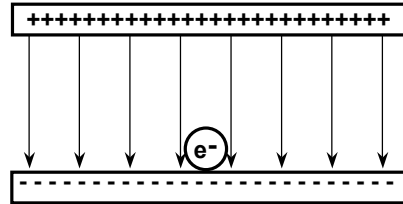
$$x = x_o + v_{ox}t$$

$$y = y_o + v_{oy}t + \frac{1}{2} at^2$$

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Motion of Charged Particles in Electric Fields

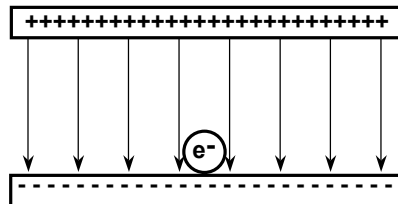
- Consider the following set up,



For an electron beginning at rest at the bottom plate, what will be its speed when it crashes into the top plate?

Spacing = 10 cm, $E = 100 \text{ N/C}$, $e = 1.6 \times 10^{-19} \text{ C}$, $m = 9.1 \times 10^{-31} \text{ kg}$

Motion of Charged Particles in Electric Fields



$$v_o = 0, y_o = 0$$

$$v_f^2 - v_o^2 = 2a\Delta x$$

Or,

$$v_f^2 = 2 \left(\frac{eE}{m} \right) \Delta x$$

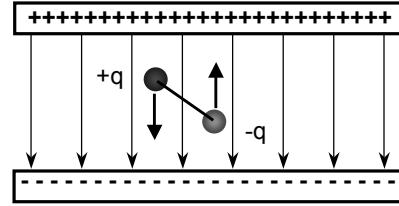
$$v_f = \sqrt{2 \left(\frac{eE}{m} \right) \Delta x}$$

$$v_f = \sqrt{2 \left(\frac{1.6 \times 10^{-19} \text{ C} \times 100 \text{ N/C}}{9.1 \times 10^{-31} \text{ kg}} \right) (0.1 \text{ m})}$$

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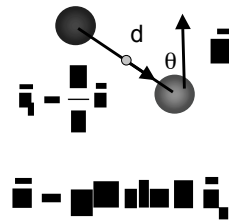
Torque on a dipole

- Force on both charges
 - 2 different direction
 - Create a torque

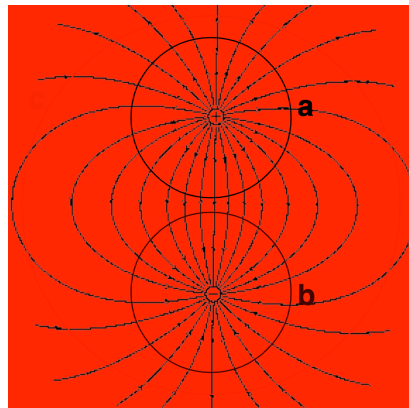


- Recall: $\vec{E} = -\nabla\phi$

- And we have



Gauss' Law



$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = q$$

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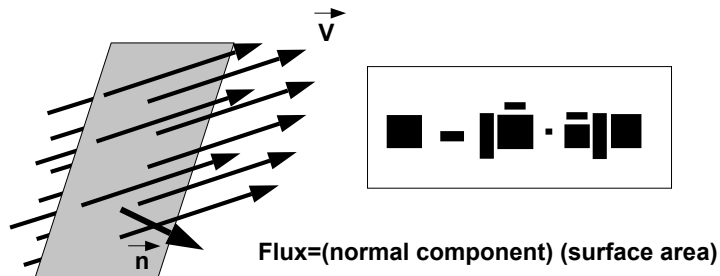
Calculating Electric Fields

- **Coulomb's Law**
Force between two point charges
Can also be used to calculate E fields

OR

- **Gauss' Law**
Relationship between Electric Fields
and charges
Uses the concept of Electric flux

Flux of a Vector Field



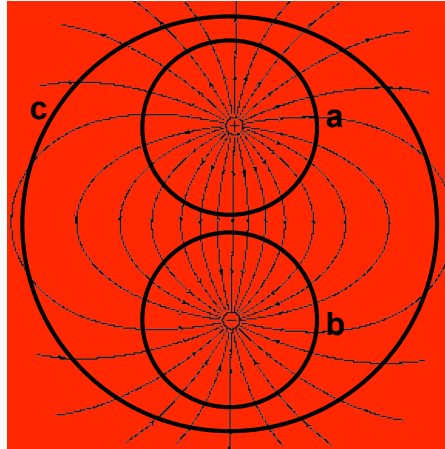
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Electric Dipole Lines of Force

Consider imaginary
spheres centered
on :

- a) +q (green)
- b) -q (red)
- c) midpoint (yellow)

- All lines leave a)
- All lines enter b)
- Equal amounts of leaving and entering lines for c)



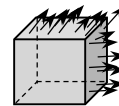
Electric Flux

- Flux:

Let's quantify previous discussion about field-line "counting"

Define: electric flux Φ_E through the closed surface S

$$\Phi_E \equiv \oint \vec{E} \cdot d\vec{S}$$

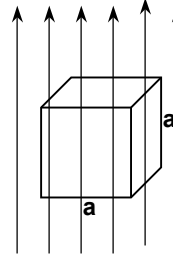


- What does this new quantity mean?
 - The integral is an integral over a CLOSED SURFACE
 - The result (the net electric flux) is a SCALAR quantity
 - $d\vec{S}$ is normal to the surface and points OUT
 - $\Rightarrow \vec{E} \cdot d\vec{S}$ uses the component of E which is NORMAL to the SURFACE
 - Therefore, the electric flux through a closed surface is the sum of the normal components of the electric field all over the surface.
 - Pay attention to the direction of the normal component as it penetrates the surface...is it "out of" or "into" the surface?
 - "Out of" is "+" "Into" is "-"

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Lecture 3, ACT 2

- Imagine a cube of side a positioned in a region of constant electric field, strength E , as shown.
 - Which of the following statements about the net electric flux Φ_E through the surface of this cube is true?

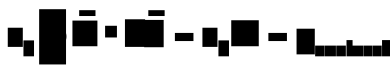


- (a) $\Phi_E = 0$ (b) $\Phi_E = 2Ea^2$ (c) $\Phi_E = 6Ea^2$

Gauss' Law



Karl Friedrich Gauss
(1777-1855)



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Gauss' Law

- Gauss' Law (a FUNDAMENTAL Law):

The net electric flux through any closed surface is proportional to the charge enclosed by that surface.

$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = \epsilon_0 \Phi = q_{\text{enclosed}}$$

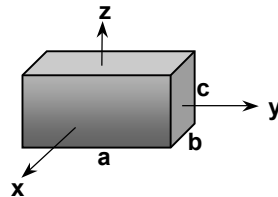
- How to Apply??

- The above eqn is TRUE always, but it doesn't look easy to use
- It is very useful in finding E when the physical situation exhibits massive SYMMETRY
- To solve the above eqn for E, you have to be able to CHOOSE a closed surface such that the integral is TRIVIAL
 - » Direction: surface must be chosen such that E is known to be either parallel or perpendicular to each piece of the surface
 - » Magnitude: surface must be chosen such that E has the same value at all points on the surface when E is perpendicular to the surface.
 - » Therefore: that allows you to bring E outside of the integral

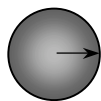
Geometry and Surface Integrals

If E is constant over a surface, and normal to it everywhere, we can take E outside the integral, leaving only a surface area

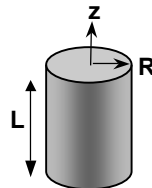
$$\oint \vec{E} \cdot d\vec{S} = E \oint dS$$



$$\oint dS = 2ac + 2bc + 2ab$$



$$\oint dS = 4\pi R^2$$

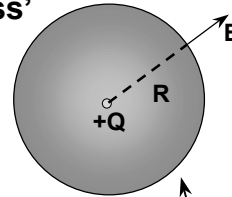


$$\oint dS = 2\pi R^2 + 2\pi RL$$

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Gauss \Rightarrow Coulomb

- We now illustrate this for the field of the point charge and prove that Gauss' Law implies Coulomb's Law.
- Symmetry \Rightarrow E field of point charge is radial and spherically symmetric
- Draw a sphere of radius R centered on the charge.



- Why?
 - E normal to every point on surface
 $\Rightarrow \vec{E} \cdot d\vec{S} = EdS$
 - E has same value at every point on surface
 \Rightarrow can take E outside of the integral!

• Therefore, $\oint \vec{E} \cdot d\vec{S} = \oint EdS = E \oint dS = 4\pi R^2 E !$

- Gauss' Law \Rightarrow \Rightarrow

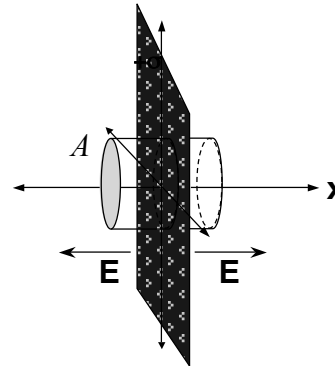
We are free to choose the surface in such problems...we call this a "Gaussian" surface



Infinite sheet of charge

- Symmetry:
direction of E = x-axis

- Therefore, CHOOSE Gaussian surface to be a cylinder whose axis is aligned with the x-axis.



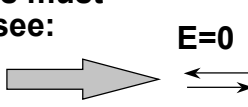
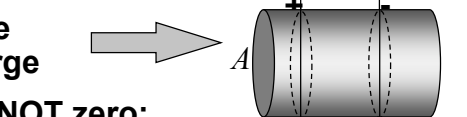

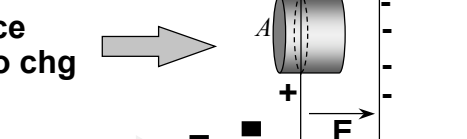
- Apply Gauss' Law:
 - On the barrel, $\oint \vec{E} \cdot d\vec{S} = 0$
 - On the ends, $\oint \vec{E} \cdot d\vec{S} = 2AE$
 - The charge enclosed = σA


Therefore, Gauss' Law $\Rightarrow \epsilon_0 (2EA) = \sigma A$ \Rightarrow

Conclusion: An infinite plane sheet of charge creates a CONSTANT electric field .

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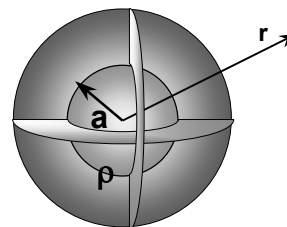
Two Infinite Sheets

- Field outside the sheets must be zero. Two ways to see:
 - Superposition 
 - Gaussian surface encloses zero charge 
- Field inside sheets is NOT zero:
 - Superposition 
 - Gaussian surface encloses non-zero chg 



Uniformly charged sphere

What is the magnitude of the electric field due to a solid sphere of radius a with uniform charge density ρ (C/m^3)?



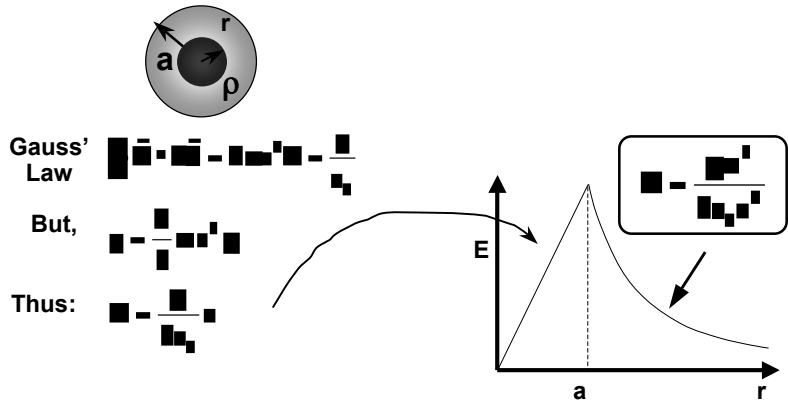
- Outside sphere: ($r > a$)
 - We have spherical symmetry centered on the center of the sphere of charge
 - Therefore, choose Gaussian surface = hollow sphere of radius r



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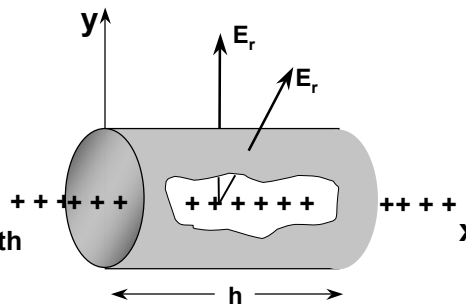
Uniformly charged sphere

- **Inside sphere: ($r < a$)**
 - We still have spherical symmetry centered on the center of the sphere of charge.
 - Therefore, choose Gaussian surface = sphere of radius r .



Infinite Line of Charge

- **Symmetry \Rightarrow E field must be \perp to line and can only depend on distance from line**
- Therefore, **CHOOSE** Gaussian surface to be a cylinder of radius r and length h aligned with the x -axis.
- **Apply Gauss' Law:**



- On the ends, $\vec{E} \cdot d\vec{S} = 0$
- On the barrel, $\oint \vec{E} \cdot d\vec{S} = 2\pi r h E$ AND $q = \lambda h \Rightarrow$

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$

NOTE: we have obtained here the same result as we did last lecture using Coulomb's Law. The symmetry makes today's derivation easier!

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Conductors & Insulators

- Consider how charge is carried on macroscopic objects.
- We will make the simplifying assumption that there are only two kinds of objects in the world:

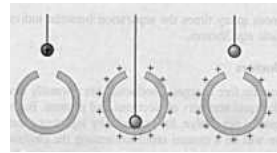
- **Insulators..** In these materials, once they are charged, the charges **ARE NOT FREE TO MOVE**. Plastics, glass, and other “bad conductors of electricity” are good examples of insulators.

- **Conductors..** In these materials, the charges **ARE FREE TO MOVE**. Metals are good examples of conductors.

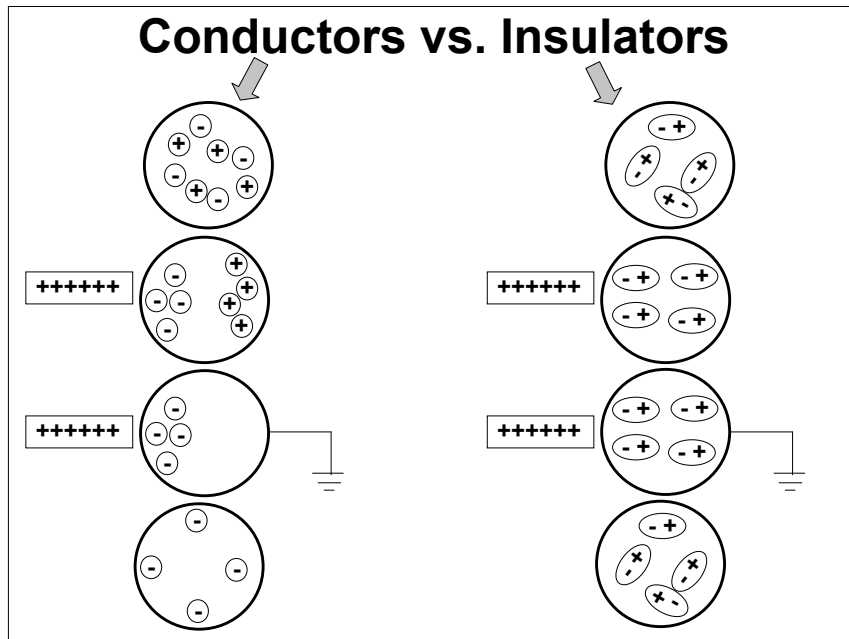
- **How do the charges move in a conductor??**

- **Hollow conducting sphere**

Charge the inside, all of this charge moves to the outside.



Conductors vs. Insulators



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Charges on a Conductor

- Why do the charges always move to the surface of a conductor ?

- Gauss' Law tells us!!
- $E = 0$ inside a conductor when in equilibrium (electrostatics) !
 - » Why?
 - If $E \neq 0$, then charges would have forces on them and they would move !

- Therefore from Gauss' Law, the charge on a conductor must only reside on the surface(s) !

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Infinite conducting
plane



Conducting
sphere