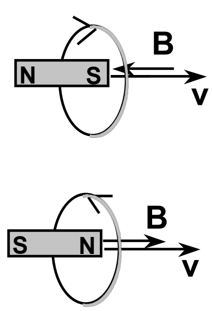
Lecture 4

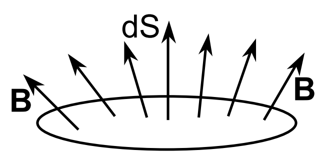
Physics 1402: Lecture 19
Today's Agenda

- **Announcements:**
  - **Midterm 1 available**
- **Homework 06 next Friday**
- **Induction**

Induction

Faraday's Law





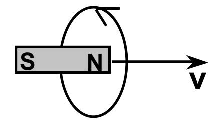
$$\Phi_B = \int \vec{B} \cdot d\vec{S}$$

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

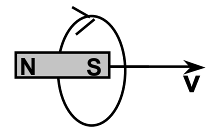
Lecture 4

Induction Effects

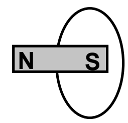
- Bar magnet moves through coil
  ⇒ Current induced in coil



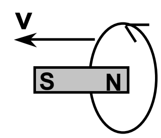
- Change pole that enters
  ⇒ Induced current changes sign



- Bar magnet stationary inside coil
  ⇒ No current induced in coil



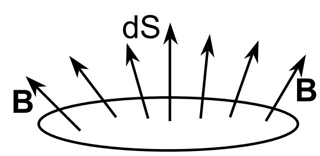
- Coil moves past fixed bar magnet
  ⇒ Current induced in coil



Faraday's Law

- Define the flux of the magnetic field through a surface (closed or open) from:

$$\Phi_B = \int \vec{B} \cdot d\vec{S}$$



- Faraday's Law:

  The emf induced around a closed circuit is determined by the time rate of change of the magnetic flux through that circuit.

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

The minus sign indicates direction of induced current (given by Lenz's Law).

Faraday''s law for many loops

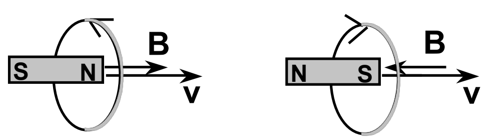
- Circuit consists of N loops:
  all same area
  Φ_B magn. flux through one loop
  loops in "series" ⇨ *emf*s add!

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Lenz's Law

- Lenz's Law:

  The induced current will appear in such a direction that it opposes the change in flux that produced it.



- Conservation of energy considerations:

  Claim: Direction of induced current must be so as to oppose the change; otherwise conservation of energy would be violated.

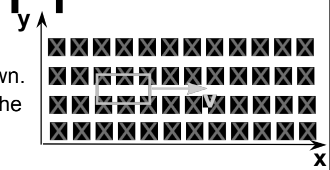
  » Why???

  - If current reinforced the change, then the change would get bigger and that would in turn induce a larger current which would increase the change, etc..

Lecture 4

Lecture 19, ACT 1

- A conducting rectangular loop moves with constant velocity v in the +x direction through a region of constant magnetic field B in the -z direction as shown.
  - What is the direction of the induced current in the loop?

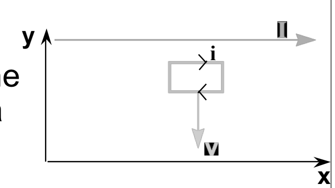


4A

(a) ccw (b) cw (c) no induced current

Lecture 19, ACT 1

- A conducting rectangular loop moves with constant velocity **v** in the **-y** direction away from a wire with a constant current **I** as shown.



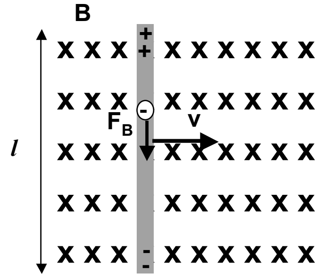
4B

- What is the direction of the induced current in the loop?

(a) ccw (b) cw (c) no induced current

Lecture 4

Motional EMF



Charges in the conductor experience the force

[illegible]

The charges will be accumulated on both ends of the conductor producing the electric field E.

The accumulation of charges will stop when the magnetic force qvB is balanced by electric force qE. Condition of equilibrium requires that

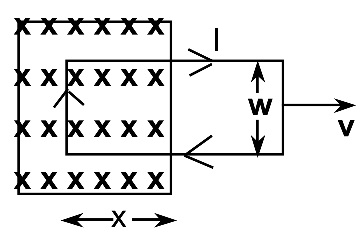
[illegible]

The electric field produced in the conductor is related to the potential difference across the ends of the conductor

[illegible]

Calculation

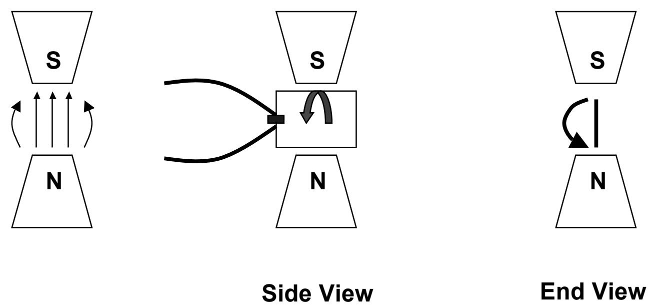
- **Suppose we pull with velocity** v **a coil of resistance** R **through a region of constant magnetic field** B**.**
  - **What will be the induced current?**
    - » **What direction?**
      - **Lenz' Law ⇒ clockwise!!**
  - **What is the magnitude?**
    - » **Magnetic Flux:** $\Phi_B = xwB$
    - » **Faraday's Law:** $\varepsilon = -\frac{d\Phi_B}{dt}$



$$\therefore \frac{d\Phi_B}{dt} = wB\frac{dx}{dt} = wBv \quad \Rightarrow \quad I = \frac{\varepsilon}{R} = \frac{wBv}{R}$$

Power Production
An Application of Faraday's Law

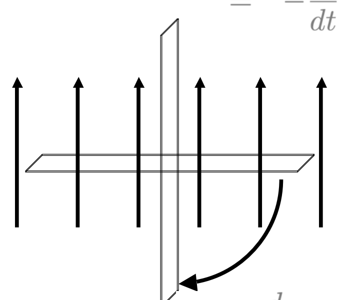
- You all know that we produce power from many sources. For example, hydroelectric power is somehow connected to the release of water from a dam. How does that work?
- Let's start by applying Faraday's Law to the following configuration:



Power Production
An Application of Faraday's Law

- Apply Faraday's Law

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S}$$
$$= -\frac{d}{dt}BA\cos\theta$$

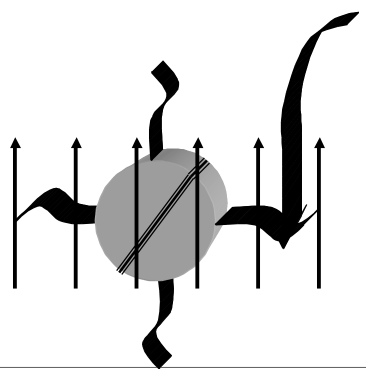


$$\varepsilon = -\frac{d}{dt}BA\cos\omega t = \omega BA\sin\omega t$$

Power Production
An Application of Faraday's Law

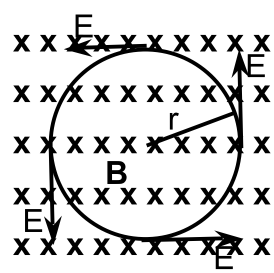
- A design schematic

$\omega t = \omega BA \sin \omega t$



ΔB → E

- Faraday's law ⇒ a changing B induces an emf which can produce a current in a loop.
- In order for charges to move (i.e., the current) there must be an electric field.
- ∴ we can state Faraday's law more generally in terms of the E field which is produced by a changing B field.



- Suppose B is increasing into the screen as shown above. An E field is induced in the direction shown. To move a charge q around the circle would require an amount of work =

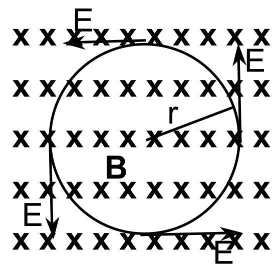
$$W = \oint q\vec{E} \cdot d\vec{l}$$

- This work can also be calculated from $\varepsilon = W/q$

ΔB → E

- **Putting these 2 eqns together:**

$$W = \oint q\vec{E}\cdot d\vec{l}$$
$$\varepsilon = \frac{W}{q}$$
$$\Rightarrow \varepsilon = \oint \vec{E}\cdot d\vec{l}$$



- **Therefore, Faraday's law can be rewritten in terms of the fields as:**

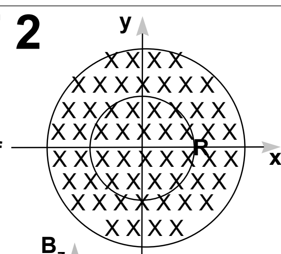
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

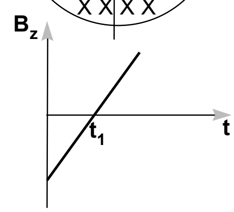
- **Note that** $\oint \vec{E}\cdot d\vec{l} = 0$ **for E fields generated by charges at rest (electrostatics) since this would correspond to the potential difference between a point and itself. Consequently, there can be no "potential function" corresponding to these induced E fields.**

Lecture 19, ACT 2

5A

- **The magnetic field in a region of space of radius 2R is aligned with the z-direction and changes in time as shown in the plot.**
  - **What is sign of the induced emf in a ring of radius R at time $t=t_1$?**





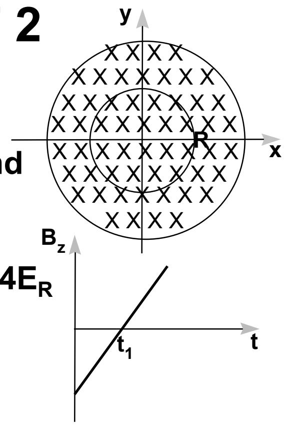
(a) $\varepsilon < 0$ (E ccw) **(b) $\varepsilon = 0$** **(c) $\varepsilon > 0$ (E cw)**

Lecture 19, ACT 2

5B

– What is the relation between the magnitudes of the induced electric fields E_R at radius R and E_{2R} at radius 2R ?

(a) $E_{2R} = E_R$ (b) $E_{2R} = 2E_R$ (c) $E_{2R} = 4E_R$

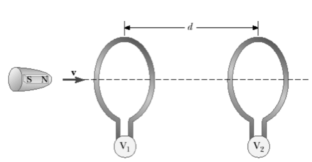


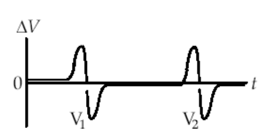
Example

An instrument based on induced emf has been used to measure projectile speeds up to 6 km/s. A small magnet is imbedded in the projectile, as shown in Figure below. The projectile passes through two coils separated by a distance ***d***. As the projectile passes through each coil a pulse of emf is induced in the coil. The time interval between pulses can be measured accurately with an oscilloscope, and thus the speed can be determined.

(a) Sketch a graph of ΔV versus ***t*** for the arrangement shown. Consider a current that flows counterclockwise as viewed from the starting point of the projectile as positive. On your graph, indicate which pulse is from coil 1 and which is from coil 2.

(b) If the pulse separation is 2.40 ms and d = 1.50 m, what is the projectile speed ?





$$v = \frac{d}{t} = \frac{1.50\ \text{m}}{2.40 \times 10^{-3}\ \text{s}} = \boxed{625\ \text{m/s}}$$

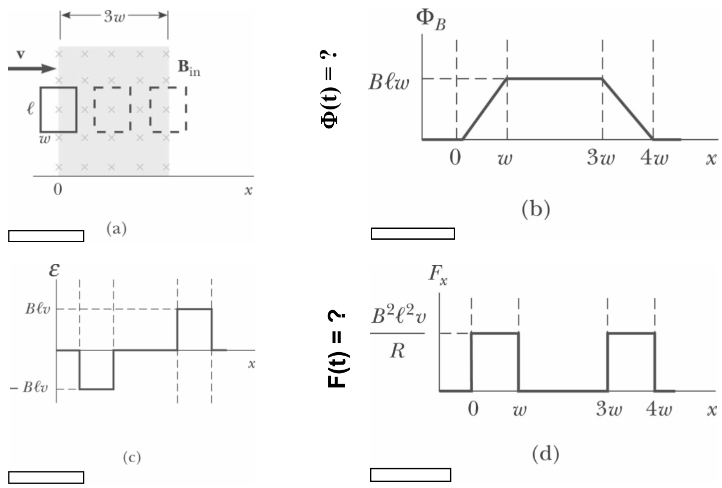
Lecture 4

A Loop Moving Through a Magnetic Field

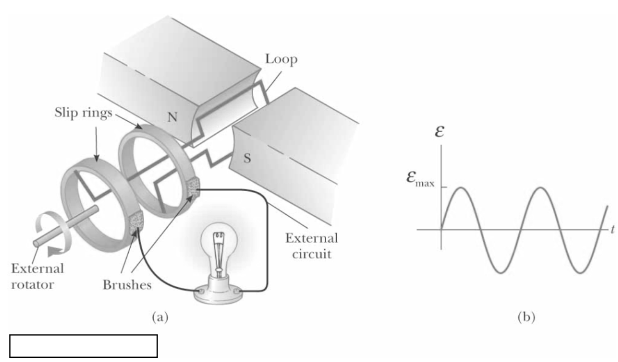
Φ(t) = ?

ε(t) = ?

F(t) = ?



Schematic Diagram of an AC Generator

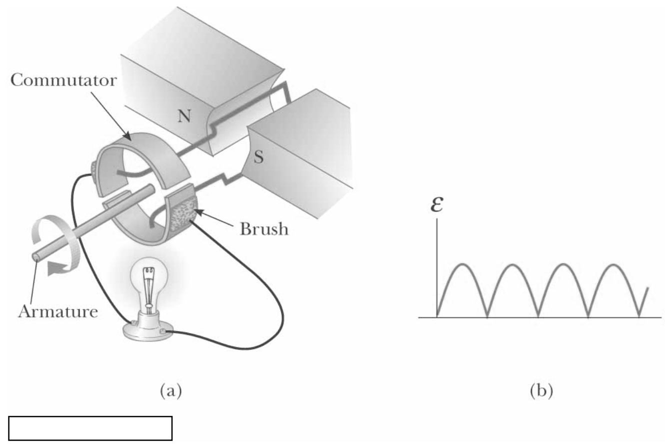


$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NAB\frac{d\,(\cos(\omega t))}{dt} = -NAB\,\omega\sin(\omega t))$$

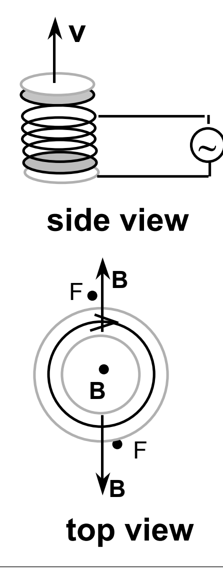
Lecture 4

Schematic Diagram of an DC Generator



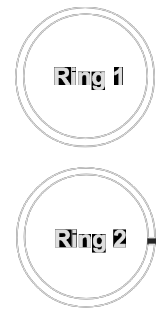
Demo
E-M Cannon

- Connect solenoid to a source of alternating voltage.
- The flux through the area ⊥ to axis of solenoid therefore changes in time.
- A conducting ring placed on top of the solenoid will have a current induced in it opposing this change.
- There will then be a force on the ring since it contains a current which is circulating in the presence of a magnetic field.



Lecture 19, ACT 3

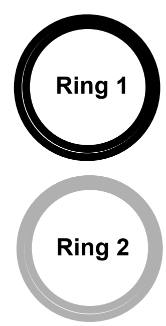
- **Suppose two aluminum rings are used in the demo; Ring 2 is identical to Ring 1 except that it has a small slit as shown. Let F_1 be the force on Ring 1; F_2 be the force on Ring 2.**



(a) $F_2 < F_1$ (b) $F_2 = F_1$ (c) $F_2 > F_1$

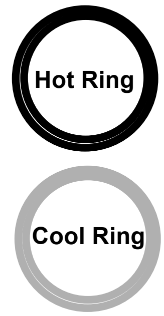
Lecture 19, ACT 4

- Suppose one copper and one aluminum rings are used in the demo; the resistance of the two rings is similar but the aluminum ring has less mass. Let $\mathbf{a}_1$ be the acceleration of ring 1 and $\mathbf{a}_2$ be the acceleration of Ring 2.



(a) $a_2 < a_1$ (b) $a_2 = a_1$ (c) $a_2 > a_1$

Lecture 19, ACT 5



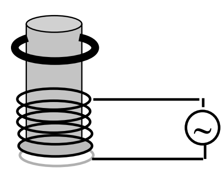
- Suppose you take the aluminum ring, shoot it off the cannon, and try to nail your annoying neighbor. Unfortunately, you just miss. You think, maybe I can hit him (her) if I change the temperature of the ring. In order to hit your neighbor, do you want to heat the ring, cool the ring, or is it just hopeless?

(a) heat **(b) cool** **(c) hopeless**

Lecture 19, ACT 6

- **Suppose the alternating magnetic field is kept at a level where the ring just levitates, but doesn't jump off. If I keep the ring suspended for about 5 minutes, is it safe to pick it up?**

(a) No **(b) Yeah, I'll do it**

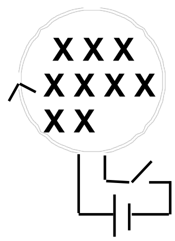


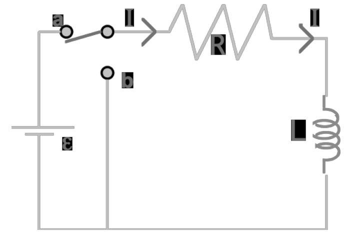
side view

Lecture 4

Induction

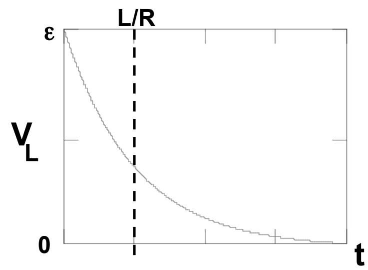
Self-Inductance, RL Circuits



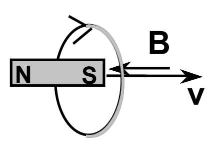


$$L \equiv \frac{\Phi_B}{I}$$

$$L \equiv -\frac{\varepsilon}{dI/dt}$$

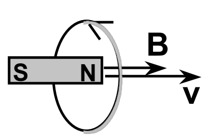


Recap from the last Chapter:
Faraday's Law of Induction



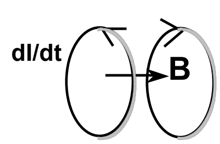
$$\epsilon = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$



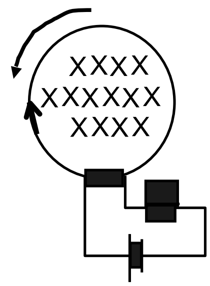
- Time dependent flux is generated by change in magnetic field strength due motion of the magnet
- Note: changing magnetic field can also be produced by time varying current in a nearby loop

Can time varying current in a conductor induce EMF in in that same conductor ?



Self-Inductance

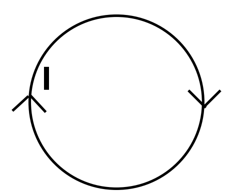
- **Consider the loop at the right.**
  - **switch closed ⇒ current starts to flow in the loop.**
  - **∴ magnetic field produced in the area enclosed by the loop.**
  - **∴ flux through loop changes**
  - **∴ emf induced in loop opposing initial emf**



- **Self-Induction: the act of a changing current through a loop inducing an opposing current in that same loop.**

Self-Inductance

- **The magnetic field produced by the current in the loop shown is proportional to that current.**



$$B \propto I$$

- **The flux, therefore, is also proportional to the current.**

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \propto I$$

- **We define this constant of proportionality between flux and current to be the inductance L.**

$$L \equiv \frac{\Phi_B}{I}$$

- **We can also define the inductance L, using Faraday's Law, in terms of the emf induced by a changing current.**

$$\epsilon = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

$$L \equiv -\frac{\epsilon}{dI/dt}$$

Self-Inductance

- **The inductance of an inductor (a set of coils in some geometry, e.g., solenoid, toroid) then, like a capacitor, can be calculated from its geometry alone if the device is constructed from conductors and air.**
- **If extra material (e.g. iron core) is added, then we need to add some knowledge of materials as we did for capacitors (dielectrics) and resistors (resistivity)**