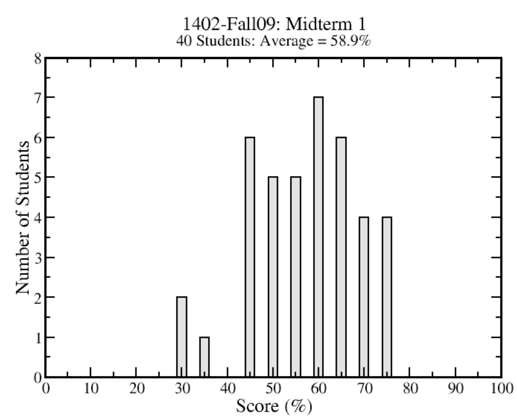
Physics 1402: Lecture 16
Today's Agenda

- Announcements:
  - Answers to midterm 1
- NO Homework due this week
- Magnetism

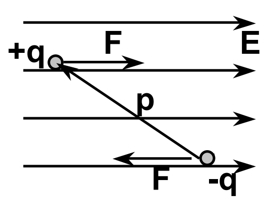
Midterm 1: Analysis

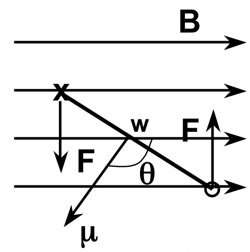
- Best: #4 with 7.2
- Worst: #3 with 4.4
- Long: 29.4/50



Lecture 4

Electric Dipole Analogy





$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{F} = q\vec{E}$$

$$\vec{p} = 2q\vec{a}$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

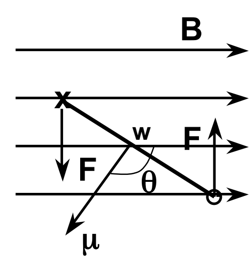
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$ **(per turn)**

$$\mu = N A I$$

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

Potential Energy of Dipole



- **Work must be done to change the orientation of a dipole (current loop) in the presence of a magnetic field.**
- **Define a potential energy U (with zero at position of max torque) corresponding to this work.**

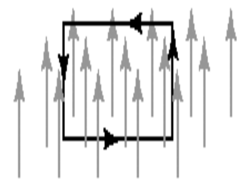
$$U = \int_{90^o}^{\theta} \tau \, d\tau \Rightarrow U = \int_{90^o}^{\theta} \mu B \sin\theta \, d\theta$$

$$U = \mu B \left[-\cos\theta\right]_{90^o}^{\theta} \Rightarrow U = -\mu B \cos\theta \Rightarrow \boxed{U = -\vec{\mu} \cdot \vec{B}}$$

Lecture 4

Lecture 16, ACT 1

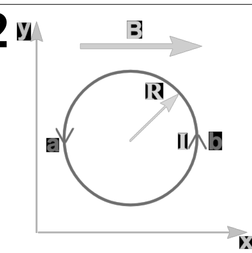
A rectangular loop is placed in a uniform magnetic field with the plane of the loop parallel to the direction of the field. If a current is made to flow through the loop in the sense shown by the arrows, the field exerts on the loop:



A) a net force. B) a net torque. C) a net force and a net torque.

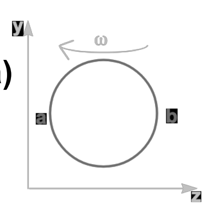
D) neither a net force nor a net torque.

Lecture 16, ACT 2

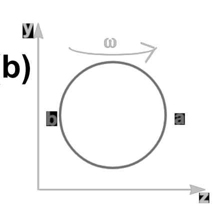


- A circular loop of radius R carries current I as shown in the diagram. A constant magnetic field B exists in the +x direction. Initially the loop is in the x-y plane.
  - The coil will rotate to which of the following positions?

(a)



(b)

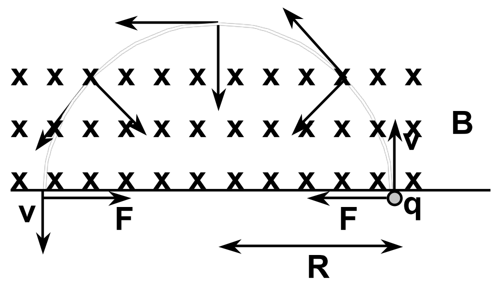


(c) It will not rotate

Lecture 4

Trajectory in Constant B Field

- Suppose charge q enters B field with velocity v as shown below. (**v**⊥**B**) What will be the path q follows?



- **Force is always ⊥ to velocity and B. What is path?**
  - **Path will be circle. F will be the centripetal force needed to keep the charge in its circular orbit. Calculate R:**

Radius of Circular Orbit

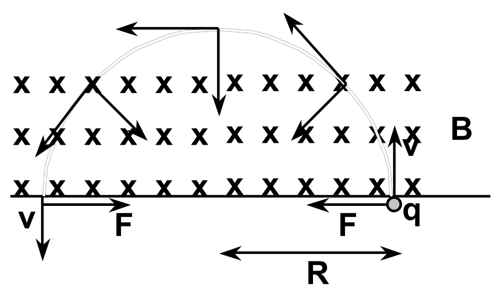
- **Lorentz force:**

$$F = qvB$$

- **centripetal acc:**

$$a = \frac{v^2}{R}$$

- **Newton's 2nd Law:**

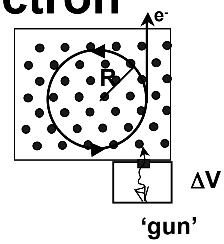


$$F = ma \quad \Rightarrow \quad qvB = m\frac{v^2}{R}$$

$$\Rightarrow \quad \boxed{R = \frac{mv}{qB}}$$

This is an important result, with useful experimental consequences !

Ratio of charge to mass for an electron



1) Turn on electron 'gun'

$$\frac{1}{2}mv^2 = qV$$

2) Turn on magnetic field B

$$R = \frac{mv}{qB}$$

3) Calculate B … next week; for now consider it a measurement

4) Rearrange in terms of measured values, V, R and B

$$v^2 = 2V\frac{q}{m} \quad \& \quad v^2 = \left(\frac{q}{m}RB\right)^2$$

$$\Rightarrow \frac{q}{m} = \frac{2V}{R^2B^2}$$

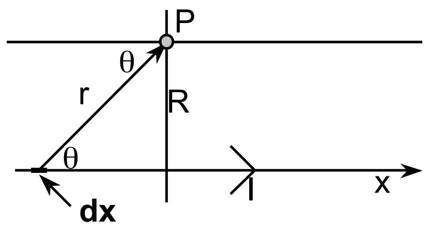
Lawrence's Insight "R cancels R"

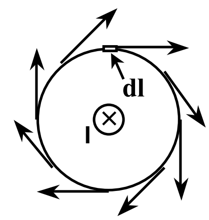
- We just derived the radius of curvature of the trajectory of a charged particle in a constant magnetic field.
- E.O. Lawrence realized in 1929 an important feature of this equation which became the basis for his invention of the cyclotron.

$$F = ma \Rightarrow qvB = m\frac{v^2}{R} \Rightarrow qBR = mv$$

- Rewrite in terms of angular velocity ω ! $\Rightarrow$ $\boxed{qBR = mR\omega}$
- R does indeed cancel R in above eqn. So What??
  - The angular velocity is independent of R!!
  - Therefore the time for one revolution is independent of the particle's energy!
  - We can write for the period, $T=2\pi/\omega$ or $T = 2\pi m/qB$
  - This is the basis for building a cyclotron.

The Laws of Biot-Savart & Ampere





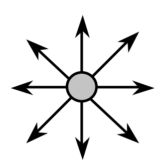
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\vec{dx} \times \vec{r}}{r^3}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Calculation of Electric Field

- **Two ways to calculate the Electric Field:**
  - **Coulomb's Law:**

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{r}$$



"Brute force"

  - **Gauss' Law**

$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = q$$

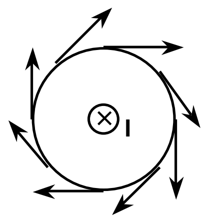
"High symmetry"

- **What are the analogous equations for the Magnetic Field?**

Calculation of Magnetic Field

- **Two ways to calculate the Magnetic Field:**
  - **Biot-Savart Law:**

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{x}\times\vec{r}}{r^3}$$



"Brute force"

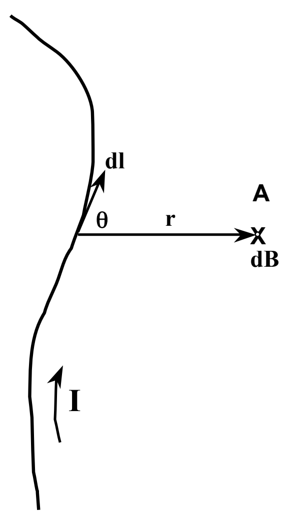
  - **Ampere's Law**

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

"High symmetry"

- **These are the analogous equations for the Magnetic Field!**

Biot-Savart Law…bits and pieces



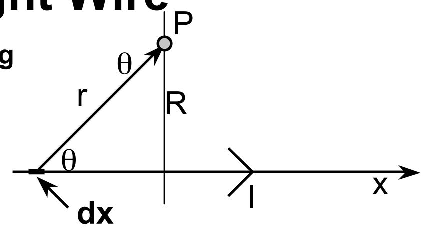
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{x}\times\vec{r}}{r^3} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$$

B in units of Tesla (T)

$\mu_0 = 4\pi \times 10^{-7}$ T m /A

So, the magnetic field "circulates" around the wire

Magnetic Field of ∞ Straight Wire



- **Calculate field at point P using Biot-Savart Law:**

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{x} \times \vec{r}}{r^3}$$

Which way is B?

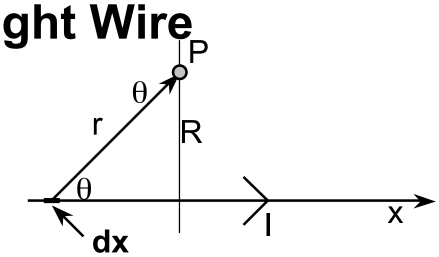
$$B = \int dB = \int_{-\infty}^{\infty} \frac{\mu_0 I}{4\pi}\frac{dx\, r\sin\theta}{r^3}$$

- **Rewrite in terms of R,θ:**

$$r = \frac{R}{\sin\theta} \quad , \quad \tan\theta = \frac{R}{-x} \quad \Rightarrow \quad x = -R\cot\theta$$

$$\therefore \quad dx = -R\left(-\frac{1}{\sin^2\theta}\right)d\theta \quad \Rightarrow \quad \frac{dx(\sin\theta)}{r^2} = \frac{\sin\theta d\theta}{R}$$

Magnetic Field of ∞ Straight Wire



$$B = \int_0^{\pi} \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}\sin\theta$$

$$B = \frac{\mu_0 I}{4\pi R}\int_0^{\pi}\sin\theta d\theta \quad \Rightarrow \quad B = \frac{\mu_0 I}{4\pi R}\left[-\cos\theta\right]_0^{\pi}$$

$$\therefore \quad \boxed{B = \frac{\mu_0 I}{2\pi R}}$$