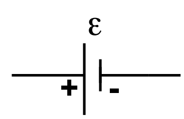
Lecture 4

Physics 1402: Lecture 10
Today's Agenda

- **Announcements:**
  - **Lectures posted on: www.phys.uconn.edu/~rcote/**
  - **HW assignments, solutions etc.**
- **Homework #3:**
  - **On Masterphysics : due Friday at 8:00 AM**
  - **Go to masteringphysics.com**

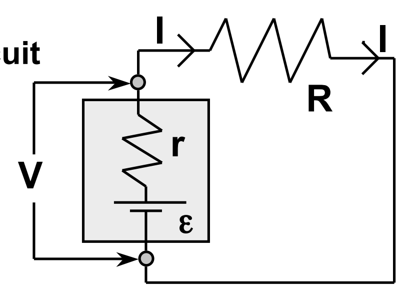
Electromotive force

- **Provides a constant potential difference between 2 points**
  - ε: "electromotive force" (emf)



- **May have an internal resistance**
  - Not "ideal" (or perfect: small loss of V)
  - Parameterized with "internal resistance" r in series with ε
- **Potential change in a circuit**

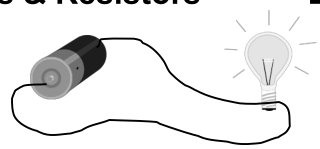
$$\varepsilon - Ir - IR = 0$$



Lecture 4

Power

Batteries & Resistors



Energy expended

chemical
to electrical
to heat

Rate is: $\frac{\text{energy}}{\text{time}} = \text{power}\left(\frac{J}{s}\right)$

What's happening?

Charges per time

Assert: $$P = VI$$

Energy "drop" per charge

For Resistors: $P = (IR)I = I^2R$ or $\frac{V^2}{R}$

Batteries (non-ideal)

- **Parameterized with "internal resistance" r in series with ε**
- **ε: "electromotive force" (emf) = V(I=0)**

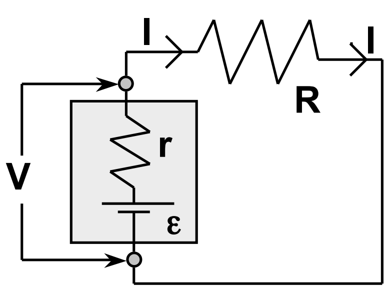
$$\varepsilon - Ir = V$$

$$\varepsilon - Ir - IR = 0$$

⬇

$$I = \frac{\varepsilon}{R + r}$$

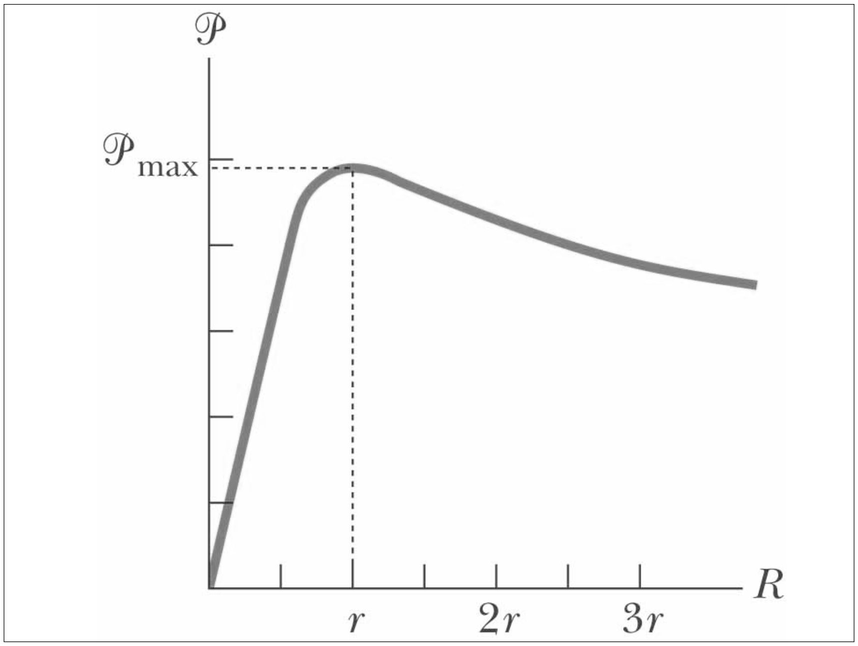
$$\Rightarrow \quad V = \varepsilon \frac{R}{R + r}$$



Power delivered to the resistor R:

$$P = I^2R = \frac{\varepsilon^2 R}{(R + r)^2}$$

P_{max} when R/r =1 !



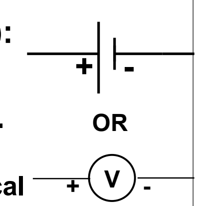
Devices

- **Conductors:**

  Purpose is to provide zero potential difference between 2 points.

  - » **Electric field is never exactly zero.. All conductors have some resistivity.**
  - » **In ordinary circuits the conductors are chosen so that their resistance is negligible.**

- **Batteries (Voltage sources, seats of emf):**

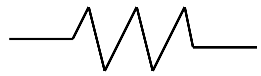
  

  Purpose is to provide a constant potential difference between 2 points.

  - » **Cannot calculate the potential difference from first principles.. electrical ↔ chemical energy conversion. Non-ideal batteries will be dealt with in terms of an "internal resistance".**

Lecture 4

Devices

- **Resistors:** 

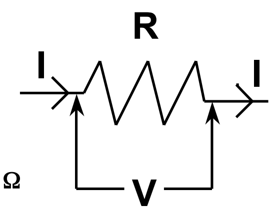
  Purpose is to limit current drawn in a circuit.
  - » **Resistance can be calculated from knowledge of the geometry of the resistor AND the "resistivity" of the material out of which it is made.**
  - » **The effective resistance of series and parallel combinations of resistors will be calculated using the concepts of potential difference and current conservation (Kirchoff's Laws).**

- **Resistance**

  Resistance is defined to be the ratio of the applied voltage to the current passing through.

  $$R \equiv \frac{V}{I}$$

  UNIT: OHM = Ω

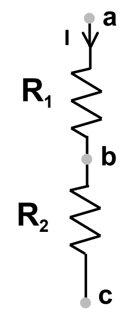


How resistance is calculated

- **Resistivity** $$\rho = \frac{E}{j}$$
  - **property of all materials**
  - **measures how much current density *j* results from a given electric field *E in that material***
  - ***units are Ohm x m (Ω m)***
- **Conductivity** $$\sigma = \frac{1}{\rho}$$
  - **sometimes used instead of resistivity**
  - **measures the same thing as ρ**

- **Resistance** $$R = \frac{\rho L}{A}$$
  - **property of an object**
  - **depends on resistivity of its material and its <u>geometry</u>**
- **Conductance** $$c = \frac{1}{R}$$
  - **sometimes used instead of resistance**
  - **measures the same thing as R**

Resistors in Series



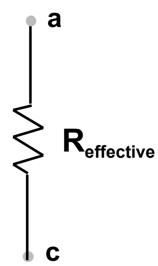
The Voltage "drops":

$$V_a - V_b = IR_1 \qquad V_b - V_c = IR_2$$

$$V_a - V_c = I(R_1 + R_2)$$

Whenever devices are in SERIES, the current is the same through both !

This reduces the circuit to:



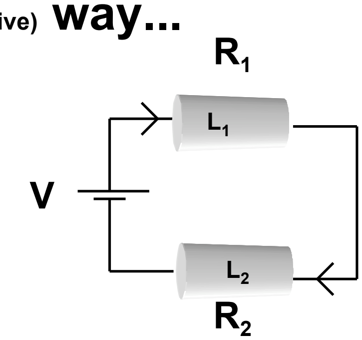
Hence: $\boxed{R_{\text{effective}} = R_1 + R_2}$

Another (intuitive) way...

Consider two cylindrical resistors with lengths L_1 and L_2

$$R_1 = \frac{\rho L_1}{A}$$

$$R_2 = \frac{\rho L_2}{A}$$



Put them together, end to end to make a longer one...

$$R_{\text{effective}} = \rho \frac{L_1 + L_2}{A} = R_1 + R_2$$

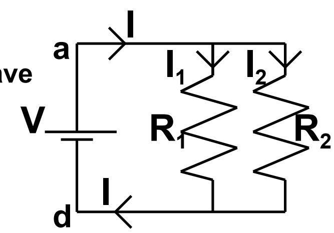
$$\boxed{R = R_1 + R_2}$$

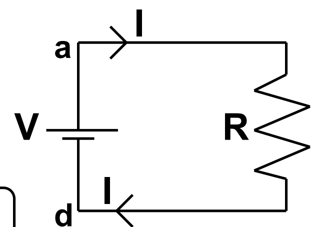
Resistors in Parallel

- What to do? $V = IR$
- Very generally, devices in parallel have the same voltage drop
- But current through R_1 is not I ! Call it I_1. Similarly, $R_2 \leftrightarrow I_2$.

$$V = I_1 R_1$$
$$V = I_2 R_2$$

- How is I related to I_1 & I_2 ?? Current is conserved!





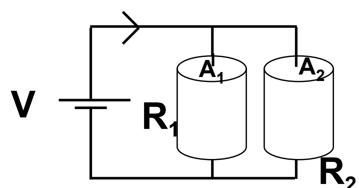
$$I = I_1 + I_2$$

$$\Rightarrow \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} \quad \Rightarrow \quad \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Another (intuitive) way...

Consider two cylindrical resistors with cross-sectional areas A_1 and A_2

$$R_1 = \frac{\rho L}{A_1}$$
$$R_2 = \frac{\rho L}{A_2}$$



Put them together, side by side … to make a "fatter" one with $A=A_1+A_2$,

$$R_{\text{effective}} = \frac{\rho L}{A_1 + A_2} \qquad \Longrightarrow \frac{1}{R_{\text{effective}}} = \frac{A_1}{\rho L} + \frac{A_2}{\rho L} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

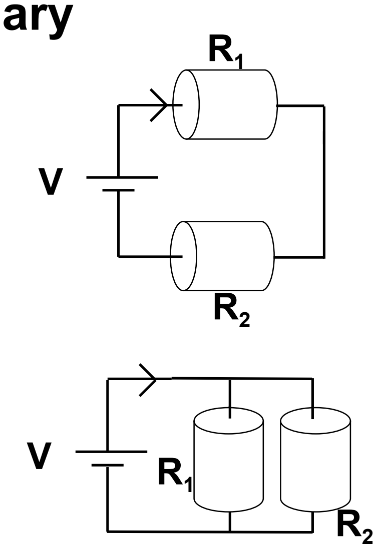
Summary

- **Resistors in series**
  - **the current is the same in both R_1 and R_2**
  - **the voltage drops add**

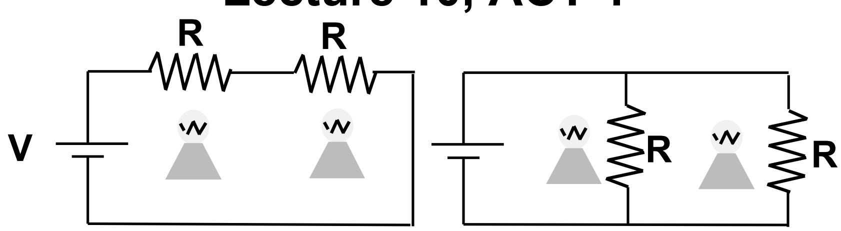
$$R = R_1 + R_2$$

- **Resistors in parallel**
  - **the voltage drop is the same in both R_1 and R_2**
  - **the currents add**

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



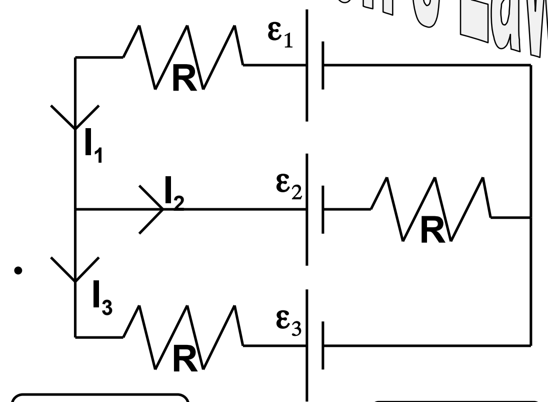
Lecture 10, ACT 1



- I have two identical light bulbs. First I hook them up in series. Then I hook them up in parallel. In which case are the bulbs brighter? (The resistors represent light bulbs whose brightness is proportional to **$P = I^2R$** through the resistor.)

A) Series B) Parallel C) The same

Kirchoff's Laws



$$\sum_{loop} V_n = 0$$

$$I_{in} = \sum I_{out}$$

Kirchoff's First Rule

"Loop Rule" or "Kirchoff's Voltage Law (KVL)"

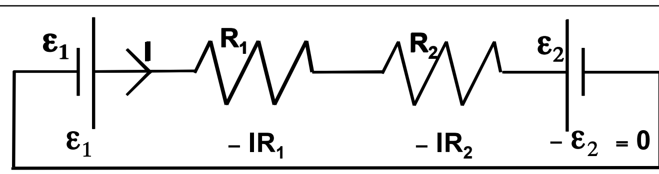
"When any closed circuit loop is traversed, the algebraic sum of the changes in potential must equal zero."

KVL: $\sum_{loop} V_n = 0$

- **This is just a restatement of what you already know: that the potential difference is independent of path!**
- **RULES OF THE ROAD:**

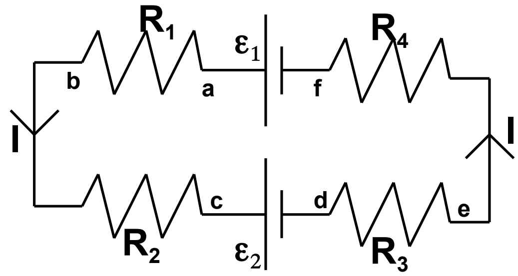
We will follow the convention that voltage gains enter with a + sign and voltage drops enter with a - sign in this equation.

Move clockwise around circuit:



$\varepsilon_1 \quad - IR_1 \quad - IR_2 \quad - \varepsilon_2 = 0$

Loop Example

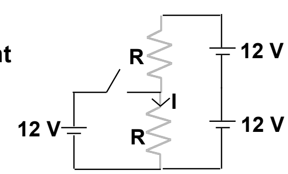


KVL: $\sum_{\text{loop}} V_n = 0 \Rightarrow -IR_1 - IR_2 - \varepsilon_2 - IR_3 - IR_4 + \varepsilon_1 = 0$

$$\Rightarrow I = \frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2 + R_3 + R_4}$$

Lecture 10, ACT 2

- **Consider the circuit shown.**
  - **The switch is initially open and the current flowing through the bottom resistor is I_0.**
  - **After the switch is closed, the current flowing through the bottom resistor is I_1.**
  - **What is the relation between I_0 and I_1?**



(a) $I_1 < I_0$ **(b) $I_1 = I_0$** **(c) $I_1 > I_0$**

Kirchoff's Second Rule

"Junction Rule" or "Kirchoff's Current Law (KCL)"

- **In deriving the formula for the equivalent resistance of 2 resistors in parallel, we applied Kirchoff's Second Rule (the junction rule).**

  "At any junction point in a circuit where the current can divide (also called a node), the sum of the currents into the node must equal the sum of the currents out of the node."

$$I_{\text{in}} = \sum I_{\text{out}}$$

- **This is just a statement of the conservation of charge at any given node.**