

Lecture 4

Physics 1402: Lecture 7 Today's Agenda

- **Announcements:**
 - Lectures posted on:
www.phys.uconn.edu/~rcote/
 - HW assignments, solutions etc.
- **Homework #2:**
 - On Masterphysics today: due Friday
 - Go to masteringphysics.com
- **Labs: Begin this week**

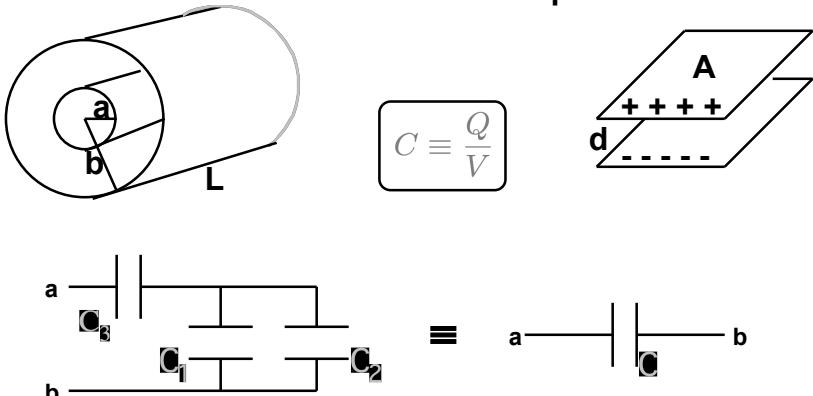
Today's Topic :

- **End Chapter 24:**
 - Definition of Capacitance
 - Example Calculations
 - (1) Parallel Plate Capacitor
 - (2) Cylindrical Capacitor
 - (3) Isolated Sphere
 - Energy stored in capacitors
 - Dielectrics
 - Capacitors in Circuits

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Capacitance


Definitions & Examples



The diagram illustrates various representations of a capacitor. At the top left is a 3D perspective of a cylindrical capacitor with length L and radii a and b . To its right is the definition equation $C \equiv \frac{Q}{V}$. Further right is a 3D perspective of a parallel plate capacitor with area A and plate separation d , showing positive charges on the top plate and negative charges on the bottom. Below these are two circuit symbols: a detailed symbol with terminals a and b and internal plates, and a simplified symbol with terminals a and b .

Capacitance

- A capacitor is a device whose purpose is to store electrical energy which can then be released in a controlled manner during a short period of time.



The diagram shows two oval-shaped conductors, one with a plus sign and one with a minus sign, representing positive and negative charges.

- A capacitor consists of 2 spatially separated conductors which can be charged to $+Q$ and $-Q$ respectively.
- The capacitance is defined as the ratio of the charge on one conductor of the capacitor to the potential difference between the conductors.

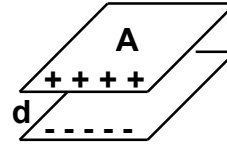
$$C \equiv \frac{Q}{V}$$

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Example 1: Parallel Plate Capacitor

- Calculate the capacitance. We assume $+\sigma$, $-\sigma$ charge densities on each plate with potential difference V :

$$C \equiv \frac{Q}{V}$$



- Need Q: $Q = \sigma A$

- Need V: from defn:

$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{\ell}$$

- Use Gauss' Law to find E

Recall: Two Infinite Sheets

(into screen)

- Field outside the sheets is zero

- Gaussian surface encloses zero net charge

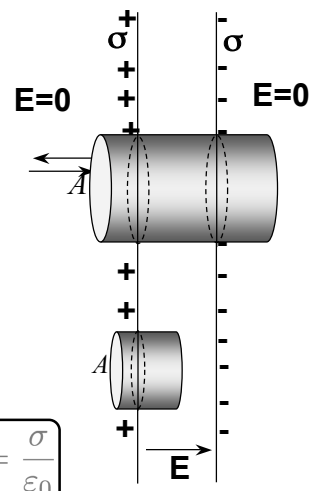
- Field inside sheets is not zero:

- Gaussian surface encloses non-zero net charge $Q = \sigma A$

$$\oint \vec{E} \cdot d\vec{S} = AE_{\text{inside}}$$



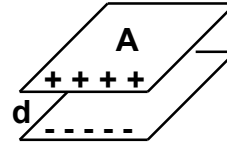
$$E = \frac{\sigma}{\epsilon_0}$$



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Example 1: Parallel Plate Capacitor

- Calculate the capacitance:
Assume +Q,-Q on plates with potential difference V.



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$
$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{\ell} = \frac{Q}{A\epsilon_0} d \quad \Rightarrow \quad C \equiv \frac{Q}{V} = \frac{A\epsilon_0}{d}$$

- As hoped for, the capacitance of this capacitor depends only on its geometry (A,d).

Dimensions of capacitance

- $C = Q/V \Rightarrow [C] = \boxed{\text{F(arad)} = \text{C/V}} = [\text{Q/V}]$

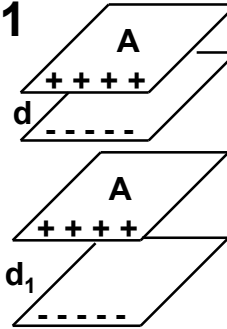
- Example: Two plates, $A = 10\text{cm} \times 10\text{cm}$
 $d = 1\text{cm}$ apart

$$\begin{aligned} \Rightarrow C &= A\epsilon_0/d = \\ &= 0.01\text{m}^2/0.01\text{m} * 8.852\text{e-}12 \text{ C}^2/\text{Jm} \\ &= 8.852\text{e-}12 \text{ F} \end{aligned}$$

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Lecture 7 - ACT 1

- Suppose the capacitor shown here is charged to Q and then the battery disconnected.
- Now suppose I pull the plates further apart so that the final separation is d_1 . $d_1 > d$



If the initial capacitance is C_0 and the final capacitance is C_1 , is ...

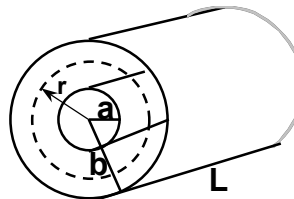
A) $C_1 > C_0$

B) $C_1 = C_0$

C) $C_1 < C_0$

Example 2: Cylindrical Capacitor

- Calculate the capacitance:
- Assume $+Q$, $-Q$ on surface of cylinders with potential difference V .



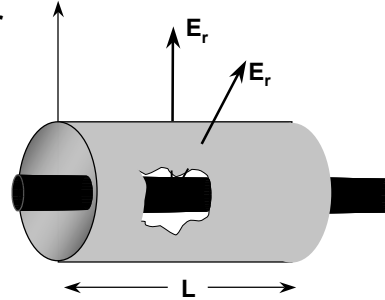
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Recall: Cylindrical Symmetry

- Gaussian surface is cylinder of radius r and length L
- Cylinder has charge Q
- Apply Gauss' Law:

$$\oint \vec{E} \cdot d\vec{S} = 2\pi r L E = \frac{Q}{\epsilon_0}$$

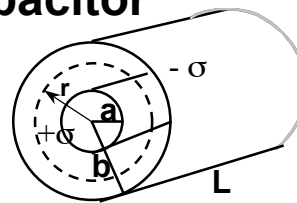
$$\Rightarrow E = \frac{Q}{2\pi\epsilon_0 L r}$$



Example 2: Cylindrical Capacitor

- Calculate the capacitance:
- Assume $+Q$, $-Q$ on surface of cylinders with potential difference V .

$$E = \frac{Q}{2\pi\epsilon_0 L r}$$



If we assume that inner cylinder has $+Q$, then the potential V is positive if we take the zero of potential to be defined at $r = b$:

$$V = - \int_b^a \vec{E} \cdot d\vec{\ell} = - \int_b^a E dr = \int_a^b \frac{Q}{2\pi\epsilon_0 L r} dr = \frac{Q}{2\pi\epsilon_0 L} \ln\left(\frac{b}{a}\right)$$

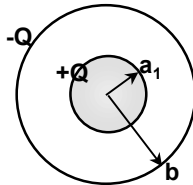
$$\Rightarrow C \equiv \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

again: depends only on system parameters (i.e., geometry)

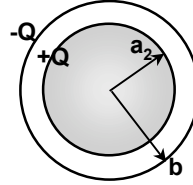
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Lecture 7, ACT 2

- In each case below, a charge of $+Q$ is placed on a solid spherical conductor and a charge of $-Q$ is placed on a concentric conducting spherical shell.
 - Let V_1 be the potential difference between the spheres with (a_1, b) .
 - Let V_2 be the potential difference between the spheres with (a_2, b) .
 - What is the relationship between V_1 and V_2 ?



(a) $V_1 < V_2$

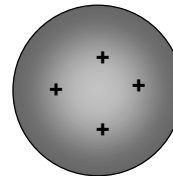


(b) $V_1 = V_2$

(c) $V_1 > V_2$

Example 3: Isolated Sphere

- Can we define the capacitance of a single isolated sphere ?
- The sphere has the ability to store a certain amount of charge at a given voltage (versus $V=0$ at infinity)



$$C \equiv \frac{Q}{\Delta V}$$

- Need ΔV : $V_\infty = 0$

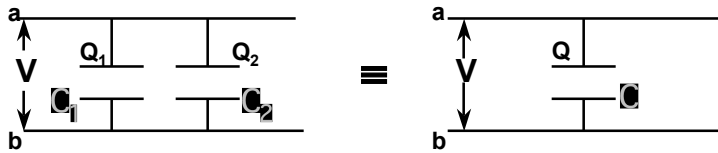
$$V_R = k_e Q/R$$

- So,

$$C = R/k_e$$

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Capacitors in Parallel



- Find “equivalent” capacitance C in the sense that no measurement at a, b could distinguish the above two situations.

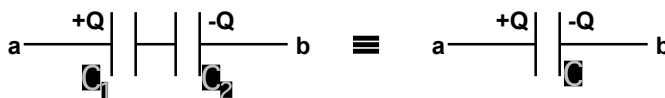
Parallel Combination: $V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$

=> Total charge: $Q = Q_1 + Q_2$

Equivalent Capacitor: $C \equiv \frac{Q}{V}$

$\Rightarrow C = C_1 + C_2$

Capacitors in Series



- Find “equivalent” capacitance C in the sense that no measurement at a, b could distinguish the above two situations.
- The charge on C_1 must be the same as the charge on C_2 since applying a potential difference across ab cannot produce a net charge on the inner plates of C_1 and C_2 .

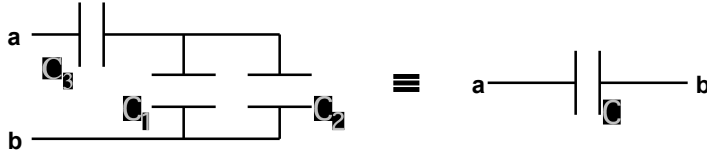
RHS: $V_{ab} = \frac{Q}{C}$

LHS: $V_{ab} = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$

$\Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

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Examples: Combinations of Capacitors



- How do we start??
 - Recognize C_3 is in series with the parallel combination on C_1 and C_2 . *i.e.*

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_1 + C_2} \Rightarrow C = \frac{C_3(C_1 + C_2)}{C_1 + C_2 + C_3}$$

Energy of a Capacitor

- How much energy is stored in a charged capacitor?
 - Calculate the work provided (usually by a battery) to charge a capacitor to +/- Q:

Calculate incremental work dW needed to add charge dq to capacitor at voltage V :

$$dW = V dq = \left(\frac{q}{C}\right) dq$$



- The total work W to charge to Q is then given by:

$$W = \frac{1}{C} \int_0^Q q dq = \frac{1}{2} \frac{Q^2}{C}$$

- In terms of the voltage V :

$$W = \frac{1}{2} CV^2$$

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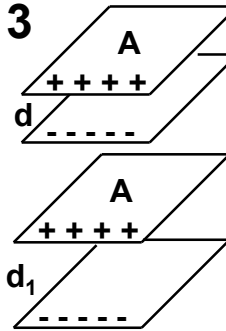
Lecture 7 – ACT 3

The same capacitor as last time.

The capacitor is charged to Q and then the battery disconnected.

Then I pull the plates further apart so that the final separation is d_1 . $d_1 > d$

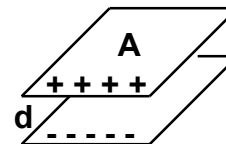
If the initial energy is U_0 and the final capacitance is U_1 , is ...



- A) $U_1 > U_0$ B) $U_1 = U_0$ C) $U_1 < U_0$

Summary

- Suppose the capacitor shown here is charged to Q and then the battery disconnected.
- Now suppose I pull the plates further apart so that the final separation is d_1 .
- How do the quantities Q , W , C , V , E change?
- **Q:** remains the same.. no way for charge to leave.
- **W:** increases.. add energy to system by separating
- **C:** decreases.. since energy \uparrow , but Q remains same
- **V:** increases.. since $C \downarrow$, but Q remains same
- **E:** remains the same.. depends only on chg density
- How much do these quantities change?.. exercise for student!!



answers:

$$W_1 = \frac{d_1}{d} W$$

$$C_1 = \frac{d}{d_1} C$$

$$V_1 = \frac{d_1}{d} V$$

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Where is the Energy Stored?

- **Claim:** energy is stored in the Electric field itself. Think of the energy needed to charge the capacitor as being the energy needed to create the field.
- To calculate the energy density in the field, first consider the constant field generated by a parallel plate capacitor:

$$W = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{Q^2}{(A\epsilon_0/d)}$$

- The Electric field is given by:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \quad \Rightarrow \quad W = \frac{1}{2} E^2 \epsilon_0 A d$$

- The energy density u in the field is given by:

$$u = \frac{W}{\text{volum}} = \frac{W}{Ad} = \frac{1}{2} \epsilon_0 E^2 \quad \text{Units: } J/m^3$$

Dielectrics

- **Empirical observation:**

Inserting a non-conducting material between the plates of a capacitor changes the VALUE of the capacitance.

- **Definition:**

The dielectric constant of a material is the ratio of the capacitance when filled with the dielectric to that without it. i.e.

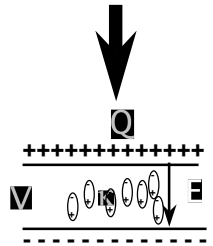
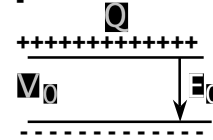
$$\kappa = \frac{C}{C_0}$$

- κ values are always > 1 (e.g., glass = 5.6; water = 78)
- They **INCREASE** the capacitance of a capacitor (generally good, since it is hard to make “big” capacitors)
- They permit more energy to be stored on a given capacitor than otherwise with vacuum (i.e., air)

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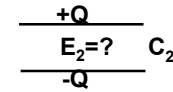
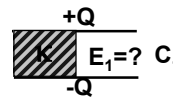
Parallel Plate Example

- Charge a parallel plate capacitor filled with vacuum (air) to potential difference V_0 .
- An amount of charge $Q = C V_0$ is deposited on each plate.
- Now insert material with dielectric constant κ .
 - Charge Q remains constant
 - Voltage decreases from V_0 to $V = \frac{V_0}{\kappa}$
 - Electric field decreases also: $E = \frac{E_0}{\kappa}$
 - So..., $C = \kappa C_0$



Lecture 7, ACT 3

- Two parallel plate capacitors are identical (same A , same d) except that C_1 has half of the space between the plates filled with a material of dielectric constant κ as shown.
- If both capacitors are given the same amount of charge Q , what is the relation between E_1 , the electric field in the air of C_1 , and E_2 , the electric field in the air of C_2



(a) $E_1 < E_2$

(b) $E_1 = E_2$

(c) $E_1 > E_2$