ELECTRICITY AND MAGNETISM

Preliminary Examination

Thursday 01/16/2014

09:00 - 12:00 in P-121

Answer a total of **THREE** questions. If you turn in excess solutions, the ones to be graded will be picked at random.

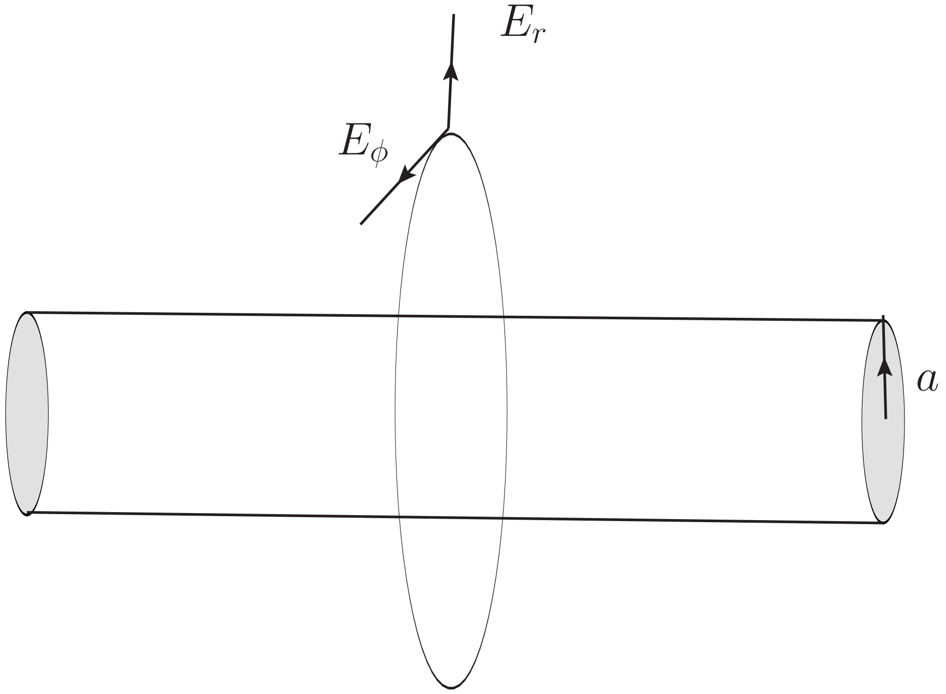
Each answer must be presented **separately** in an answer book or on sheets of paper stapled together. Make sure you clearly indicate who you are and what is the problem you are solving on each book/sheet of paper. Double-check that you include everything you want graded, and nothing else.

You are allowed to use a result stated in one part of a problem in the subsequent parts even if you cannot derive it. On the last page you will find some potentially useful formulas.

Problem 1. An infinitely long conducting cylinder of radius a has a surface charge density $\sigma(\phi)$, where ϕ is the polar angle relative to the axis of the cylinder. The total charge on the surface of the cylinder vanishes. The cylinder is surrounded by a dielectric medium with electric permittivity ϵ_d that contains no free charges. The tangential component of the electric field at $r \geq a$ is given by $E_\phi = -\kappa \cos\phi \frac{r-a}{r^3}$, where κ is a constant.

(a) Find the radial component of the electric field inside the dielectric.

(b) Find the surface charge $\sigma(\phi)$ on the conductor.



Problem 2. A charge-free conducting, isotropic medium has an electric permittivity ϵ, magnetic permeability μ and conductivity σ.

(a) Using Maxwell's equations, derive an expression for the phase velocity for monochromatic E and B fields of fixed polarization, propagating through the medium as plane waves of frequency ω.

(b) How does the phase velocity depend on frequency, electric permittivity and magnetic permeability? Discuss the limits of poor conductor ($\sigma \ll \omega\epsilon$) and a good conductor ($\sigma \gg \omega\epsilon$).

(c) Derive an expression for the attenuation of the waves. Under what conditions are E and B in phase?

Problem 3. Two long cylindrical conductors of radius a are parallel and separated by a distance $d \gg a$.

(a) Show that the capacitance per unit length is given approximately by $C \approx \frac{1}{4\ln(d/a)}$.

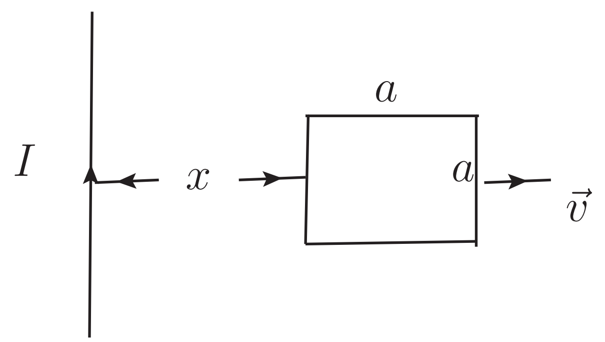
(b) What is the energy per unit length stored in the system if the two conductors are kept at the potential difference V?

Problem 4. A long wire carries a steady current I. Nearby to the long wire is a square loop of wire (side length a) with resistance R. A force is applied on the loop away from the wire so that the loop maintains a constant velocity v. Assume that the magnetic field produced by current in the loop is small compared to the field produced by the wire.

(a) At the moment when the left edge is at a distance x away from the wire, find the magnetic flux through the loop and the magnitude of the current circulating around the loop.

(b) Determine the magnetic force on the loop and the power one needs to supply to keep the loop moving at a constant velocity, as a function of position x. Show that this power is always positive, independent of whether the loop is pushed towards, or away from the wire.

(c) Qualitatively, what in the above parts would change if the velocity $\vec{v}$ were parallel to the current I instead of perpendicular to it?



Standard vector operations in three common coordinate systems

Cartesian coordinates x, y, z

$$\begin{aligned}
\nabla &= \hat{\mathbf{e}}_x \frac{\partial}{\partial x} + \hat{\mathbf{e}}_y \frac{\partial}{\partial y} + \hat{\mathbf{e}}_z \frac{\partial}{\partial z} \\
\nabla \cdot \mathbf{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\
\nabla \times \mathbf{A} &= \hat{\mathbf{e}}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{\mathbf{e}}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{\mathbf{e}}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\
\nabla^2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}
\end{aligned}$$

cylindrical coordinates ρ, ϕ, z

$$\begin{aligned}
\nabla &= \hat{\mathbf{e}}_\rho \frac{\partial}{\partial \rho} + \hat{\mathbf{e}}_\phi \frac{1}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{e}}_z \frac{\partial}{\partial z} \\
\nabla \cdot \mathbf{A} &= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \\
\nabla \times \mathbf{A} &= \hat{\mathbf{e}}_\rho \left[\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] + \hat{\mathbf{e}}_\phi \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right] + \hat{\mathbf{e}}_z \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi} \right] \\
\nabla^2 &= \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}
\end{aligned}$$

spherical polar coordinates r, θ, ϕ

$$\begin{aligned}
\nabla &= \hat{\mathbf{e}}_r \frac{\partial}{\partial r} + \hat{\mathbf{e}}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\mathbf{e}}_\phi \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi} \\
\nabla \cdot \mathbf{A} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta A_\theta) + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi} \\
\nabla \times \mathbf{A} &= \hat{\mathbf{e}}_r \frac{1}{r \sin\theta} \left[\frac{\partial}{\partial \theta} (\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial \phi} \right] + \hat{\mathbf{e}}_\theta \left[\frac{1}{r \sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right] + \hat{\mathbf{e}}_\phi \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \\
\nabla^2 &= \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \sin\theta \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2}{\partial \phi^2} \\
&\left[\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} \equiv \frac{1}{r} \frac{\partial^2}{\partial r^2} r \right]
\end{aligned}$$