CLASSICAL MECHANICS/ ELECTRICITY AND MAGNETISM

Preliminary Examination

January 14, 2009

9:00 - 15:00 in P-121

Answer a total of **SIX** questions, choosing **THREE** from each section. If you turn in excess solutions, the ones to be graded will be picked at random.

Each answer must be presented **separately** in an answer book. Make sure you clearly indicate who you are, and the problem you are answering. Double-check that you include everything you want graded, and nothing else.

SECTION A - Classical Mechanics

A1. Consider a two dimensional mechanical system with the Hamiltonian

$$H = \frac{1}{2m}[p_1^2 + p_2^2] - 2p_1 x_1 x_2 - p_2 x_1^2 + \frac{1}{2}k(x_1^2 + x_2^2) + m[2x_1^2 x_2^2 + \frac{1}{2}x_1^4] \tag{1}$$

(a) Find the transformation which transforms this into a Hamiltonian of a two dimensional harmonic oscillator. Show that this transformation is canonical, that is, it does not change the Poisson brackets of the canonical variables.

(b) Find the general solution of the equations of motion of the system.

A2. Consider a solid cylinder of mass m and radius r sliding without rolling down the smooth incline of mass M that is free to move on a horizontal plane without friction as shown in Figure 1.

(a) How far has the incline moved by the time the cylinder has descended from the rest a vertical distance h?

(b) Now suppose that the cylinder is free to roll down the incline without slipping. How far does the incline move in this case?

(c) In which case does the cylinder reach the bottom faster? How does your answer depend on the radius of the cylinder?

A3. As shown in Figure 2, three point-like masses (two of them equal) and massless springs (spring constant k) connecting them are constrained to move in a frictionless tube of radius R. This system is in a gravitational field (g). The springs are of zero length at equilibrium and the masses may move through one another. Using the Lagrangian method, find normal modes of small vibrations about the position of equilibrium of the system and describe each of the modes.

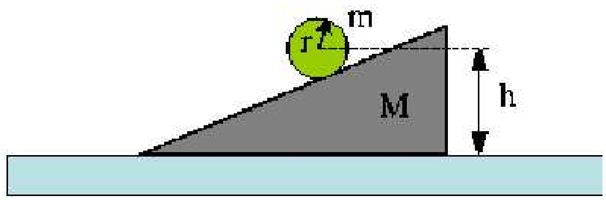

Figure 1: For problem **A2**.

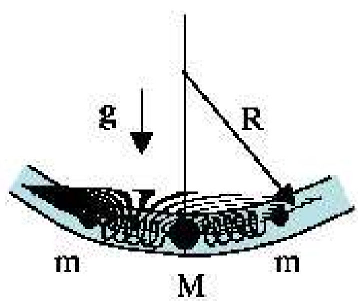

Figure 2: For problem **A3**.

A4. A so-called "light clock" is shown in the Figure. It consists of a flashtube and photocell arranged side-by-side, but separated by a screen so they see each other only through mirror M, positioned a distance d away as shown. When the photocell responds to a flash, the flashtube is triggered with negligible delay and emits a flash towards M: The clock "ticks" every $2d/c$ seconds in its rest frame.

(a) If the clock is traveling with speed v relative to an observer, in a direction perpendicular to the line formed by the flashtube/photocell and mirror, show by geometric or algebraic construction that the observer sees relativistic time dilation as the clock moves by, that is $\Delta t = \Delta t'\gamma$, where $\Delta t'$ is the time of one tick in the clock's rest frame, and $\gamma = 1/\sqrt{1 - v^2/c^2}$.

(b) If the motion is in a direction *parallel* to the clock arrangement, show that the same dilation is observed.

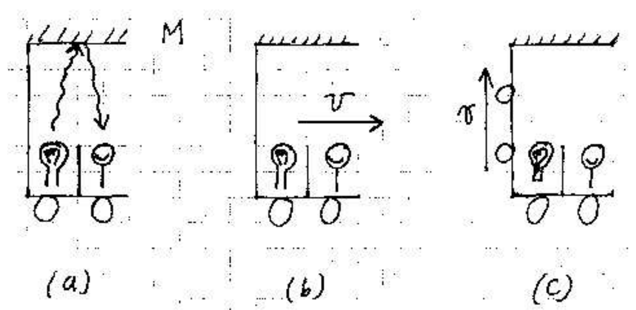

Figure 3: Light clock. (a) at rest, (b) moving as shown, and (c) moving in a perpendicular direction relative to (b).

SECTION B - Electricity and Magnetism

B1. A point magnetic dipole m in vacuum (medium 1) is pointing toward the plane surface of a medium with permeability μ (medium 2). The distance between the dipole and the surface is d, as shown in Figure 4.

(a) Solve for the magnetic field B within the medium.

(b) What is the force acting on the dipole?

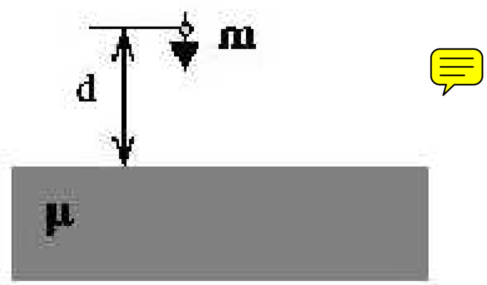

Figure 4: For problem **B1**.

B2. The plane $z = 0$ carries surface charge density

$$\sigma(x, y) = \sigma_0 \cos(ax + by).$$

Find electrostatic potential in all space (for $z > 0$ and $z < 0$).

B3. A transmission line consisting of two concentric circular cylinders of metal with conductivity σ and skin depth δ is filled with a uniform, lossless, dielectric (μ, ϵ). A TEM mode is propagated along this line.

(a) Show the time-averaged power delivered along the line is

$$P = \sqrt{\frac{\mu}{\epsilon}}\pi a^2 |H_0|^2 \ln\frac{b}{a}$$

where $|H_0|$ is the peak values of the azimuthal magnetic field at the sorface of the inner conductor.

(b) Show that the trasmitted power is attenuated along the line as

$$\begin{aligned} P &= P_0 e^{-2\gamma z} \\ \gamma &= \frac{1}{2\sigma\delta}\sqrt{\frac{\epsilon}{\mu}}\frac{\frac{1}{a}+\frac{1}{b}}{\ln\frac{b}{a}}. \end{aligned}$$

B4. Consider a circularly polarized wave with

$$\begin{aligned} \vec{E}(x,y,x,t) &= \left[E_0(x,y)(\hat{x}\pm i\hat{y}) + \frac{i}{k}\left(\frac{\partial E_0}{\partial x} \pm i\frac{\partial E_0}{\partial y}\right)\hat{z}\right] e^{ikz-i\omega t} \\ E_0(x,y) &= E_0 e^{-(x^2+y^2)/2\sigma} \\ \vec{B} &= \mp i\sqrt{\mu\epsilon}\vec{E}. \end{aligned}$$

Find the energy and angular momentum carried by the wave. What is their ratio? Interpret your result in terms of quanta of radiation.